50074

UNITED NATIONS
ECONOMIC and SOCIAL COUNCIL

Distr.
LIMITED

E/CN.14/NAC/46/Rev.1
2 August 1971

Original: ENGLISH

ECONOMIC COMMISSION FOR AFRICA
Working Group on Production Accounts
Commodity Balances and Intput-Output
Analysis at both Current and Constant prices
Dakar, 4 - 12 October 1971

THE OBJECTIVES, CONCEPTS AND METHODS OF INPUT-OUTPUT ANALYSIS

(Document prepared by the U.N. Statistical Office, New York)

071-629

THE OBJECTIVES, CONCEPTS AND METHODS OF INPUT-OUTPUT ANALYSIS

I. INTRODUCTION

1. Input-output tables and national accounts have been developed separately, despite their strong theoretical links. In some countries, they are compiled by different institutions and the information they provide is not necessarily consistent. The reason for the distinct development of the two schemes is the different types of uses to which each of them have been applied. National accounts were developed as a consistent bookkeeping scheme, containing data which could be used for any type of economic analysis or planning. Input-output tables, however, were created and developed, mainly for use in a special type of economic analysis, summarized in the input-output theory.

2. This paper deals with the considerations which are at the basis of the structure of the input-output tables which may be developed on the basis of the SNA. Chapter II explains the theoretical link between input-output tables and the SNA. In chapter III, input-output theory and some of its applications are discussed. Chapter IV reviews a number of factors which can affect the stability of the input-output coefficients and the recommendations made in the SNA to eliminate some of these possible disturbances. The effect of each of the recommendations on the structure of the input-output tabless is illustrated on the basis of a numerical example derived from the SNA - matrix (table 2.1 of the SNA). Chapter V indicates which consequences these recommendations have for the input-output techniques developed in chapter III.

II. NATIONAL ACCOUNTS AND INPUT-OUTPUT TABLES

A. The links between the two systems

3. National accounts can be defined as a system for registration of transactions, or flows, in goods and services, financial claims and assets and transfers, which are generated by distinct transformation processes. Each transformation process is represented by a separate account in which incoming and outgoing flows are registered.

4. Two basic types of transformation processes may be distinguished:

(i) Technical or production transformation processes, which relate to technical relationships such as the production function which can take various forms.

(ii) Non-technical or income spending transformation processes, which refer to the patterns of spending the income which is formed in the production processes. The flows or transactions involved here are determined either by relations of a behaviouristic nature such as the consumption function and the considerations determining the composition of portfolios, or they are of an institutional character, such as the tax-income and expenditure functions.

5. Each of these two types of transformation processes can be subdivided into separate parts, according to the different relationships between the flows involved. The more the transformation processes and associated flows are aggregated, the less separate relationships can be identified. The flows will be more heterogeneous and the relations between them less stable over time. This point is of considerable importance for the structure of the input-output table.

6. In the SNA, the technical transformation process is represented by the Production accounts and the non-technical ones by the Consumption and the Accumulation accounts. A fourth account, the Rest of the World account, deals with these three accounts for transactions of the country under consideration with all other countries. The four accounts and some of the more important flows between them are presented in matric form in table 1.

Table 1. The consolidated SNA matrix in simplified form, showing the main classes of transactions

	Production	Consumption	Accumulation	Rest of the world
Production	Intermediate consumption production	Consumption	Gross capital formation	Exports - Imports
Consumption	Value added			Compensation of employees and property income from abroad, net Current transfers from abroad, net
Accumulation		Savings		Capital transfers from abroad, net Net borrowing from abroad
Rest of the world				

7. In table 1, each flow is only registered once, namely at the intersection of the column which represents the account from which the flow leaves, and the row which stands for the account into which it enters. For instance, the flow of consumption expenditures is registered at the intersection of the production row (incoming money flow) and the consumption column (outgoing money flow). Value added is an incoming flow into the consumption account and an outgoing flow in the production account, and is consequently registered at the intersection of the production column and the consumption row. The location of the other flows can easily be verified by the reader. The rows and columns of the matrix correspond to the debit and credit sides, respectively, of the corresponding accounts. The row and column totals for any particular account are equal.

8. As in table 1, the diagonal matrix elements may register flows between transformation processes in the same group, or these flows may be netted out. Whichever procedure is followed, row and column totals remain equal because any amount registered in a diagonal element increases the row and column totals with equal amounts. The consequences of a subdivision of one of the transformation processes is that some flows, which were internal to the process and could be netted out against each other, now concern different sub-transformation processes.

9. The only diagonal element retained in table 1 is that of the production account which represents the deliveries from one production process to another. This is done because the diagonal element is of special interest for input-output analysis. To focus the attention on this element, the rows and columns of the consumption, accumulation and rest of the world accounts are, in table 2, restricted to the production of industries, i.e. of commodities, and the supply of commodities from imports and the uses to which the supply of these commodities are put. Industries are producers which usually sell their gross output of goods and services in the market at a price which is intended to cover their costs of production and commodities are goods and services which are usually sold in the market at this price.

Table 2. The production matrix

	Production	Final demand	
Pro-duction	Intermediate consumption	Final consumption + gross capital formation + exports - imports	Gross output
Primary inputs	Value added		
	Gross input		

10. The diagonal element of the production account is retained in table 2. The next step is to subdivide the production process by industries, as shown in table 3.

Table 3. Production matrix for n industries

		Production	Final demand					
		Industries	Con-sumption	Gross capital formation	Exports	Total	Minus Imports	Gross output
Production	Industries	X_{11} X_{12} . . . . . . X_{1n}	C_1	I_1	E_1	F_1	$-M_1$	P_1
		X_{21} X_{22} . . . . . . X_{2n}	C_2	I_2	E_2	F_2	$-M_2$	P_2
		. . . .	. . . .	. . . .	. . .	. . .	. . .	. . .
		X_{n1} X_{n2} X_{nn}	C_n	I_n	E_n	F_n	$-M_n$	P_n
Primary inputs	Value added	Y_1 Y_2 . . . . . . Y_n						
	Im-ports							
Gross input		P_1 P_2 . . . . . . P_n						

By subdividing the production row and column in this way, the total amount of intermediate deliveries, which was internal before is split in intermediate deliveries from one production account to another. The other elements of the rows and columns which concern industries also have to be split. The commodities entering into

intermediate and final uses are classified according to the industry where the commodities are characteristically produced; and their cost to the users (purchasers' values) is divided into the value at the establishment of the producer (producers' values) and trade and transport margins. Value added is classified by receiving industry. Imported commodities are classified according to the industries in which they are characteristically produced. In the table they are shown as negative items in a separate column. Thus, all entries in the matrix of intermediate consumption (X_{ij}) and in the vectors of final demand (C_i, I_i, E_i, F_i) include imported, as well as domestically produced commodities. Consequently, in order to arrive at an equality between gross output per industry and the sum of the corresponding row elements in table 3, it is necessary to subtract imported commodities that are characteristically produced by the industry. In symbolic form:

$$\sum X_{ij} + (C_i + I_i + E_i) - M_i = P_i$$

Definition of the symbols:

C_i - expenditures on consumption of commodities characteristically produced by industry i.

E_i - value of exported commodities characteristically produced by industry i.

F_i - final demand for commodities characteristically produced by industry i; ($F_i = C_i + I_i + E_i$).

I_i - expenditures on gross capital formation in commodities characteristically produced by industry i.

M_i - imported commodities characteristically produced by industry i

P_i - gross output of industry i.

X_{ij} - intermediate delivery of commodities to industry j, which are characteristically produced by industry i.

Y_i - value added of industry i.

The number of industries, distinguished is n, so i = 1, 2, ... n.

11. It should be emphasized that table 3 illustrates one class of input-output table only, namely, industry x industry tables in producers' values. As we shall see later in this paper, there are a number of classes of input-output tables, for example, industry x industry tables in approximate basic values or in purchasers' values, and commodity x commodity tables in each of the preceding types of valuation. Industry x commodity tables in these forms of valuation are also prepared.

12. The following matrices and vectors are represented in table 3:

(i) the input-output matrix

$$\begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1n} \\ X_{21} & X_{22} & \cdots & X_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ X_{n1} & X_{n2} & \cdots & X_{nn} \end{bmatrix}$$

(ii) the matrix of final demand:

$$\begin{bmatrix} C_1 & I_1 & E_1 \\ C_2 & I_2 & E_2 \\ \cdots & \cdots & \cdots \\ C_n & I_n & E_n \end{bmatrix}$$

(iii) the row vector of value added: $(Y_1 \quad Y_2 \quad \cdots \quad Y_n)$

(iv) the column vectors of

- consumption $\begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix}$

- gross capital formation $\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}$

- exports $\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$

- total final demand $\begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{bmatrix}$

(v) the negative column vector of imports $\begin{bmatrix} -M_1 \\ -M_2 \\ -M_n \end{bmatrix}$

(vi) the column vector of gross output $\begin{bmatrix} P_1 \\ P_2 \\ . \\ P_n \end{bmatrix}$

(vii) the row vector of gross input $(P_1 \quad P_2 \; . \; . \; . \; . \; P_n)$

B. A numerical example

13. To illustrate the argument in the following chapters, the numerical example shown in table 4 will be used. The example is derived from the SNA matrix. Four industry groups are distinguished: (1) Agriculture and mining, (2) Manufacturing and construction, (3) Other industries (mainly services) except transportation and distribution (4) Transportation and distribution. In addition, three final demand categories are distinguished, namely consumption, gross capital formation and exports.

Table 4. Production matrix for four industries

			Production				Final demand				
			Industries								
			1	2	3	4	Con-sumption	Gross capital formation	Exports	Minus Imports	Total
Production	Industries	1	1	27	3		11		1	-23	20
		2	6	121	9	12	89	42	33	-35	277
		3	1	14	4	3	40	3	5	- 2	68
		4	1	13	2	10	39	2	11	- 5	73
Primary inputs	Value added		11	102	50	48					211
	Im-ports										
Total			20	277	68	73	179	47	50	-65	

III. INPUT-OUTPUT ANALYSIS AND ITS APPLICATIONS

A. The basic assumptions

14. Input-output analysis examines the interrelationships between the output of each industry and each of its inputs. From these interrelationships, relations between final demand and gross output and between final demand and the inputs of each industry can be derived which are of interest in economic analysis and planning.

15. Input-output analysis is based on three types of assumption with regard to the structure of the input-output matrix. In the first place it assumes that a fixed ratio exists between the gross output of each industry and its inputs, i.e., that the input-output coefficient ($a_{ji} = \frac{X_{ji}}{P_i}$) is constant.[1/] (III.1). Although not absolutely necessary in input-output analysis, this assumption is sometimes extended to value added, i.e., the value added coefficient: $Y_i = \frac{Y_i}{P_i}$ (III.2) is assumed to be constant as well.

16. The second assumption of input-output analysis is that each industry only produces one type of product, called the characteristic product of this industry. The third assumption concerns the uniformity of output and input flows. For instance, in the case of cars it is assumed that only one uniform model is produced, and the input flow of metal sheets into car production is assumed to be of uniform quality. The second and third assumptions support the first assumption of constant input-output coefficients.

B. The derivation of input-output coefficients

17. In equations (III.1) and (III.2) a_{ji} and y_i were assumed to be constant. This makes it possible to express the input flows in terms of the gross output of industries. This gives: $X_{ji} = a_{ji} \; P_i$ (III.1)[1/]; $Y_i = y_i \; P_i$ (III.2)[1/]. These expressions may be substituted for the input flows in the production matrix of table 4, as shown in table 5 below.

1/ j indicates elements in the rows and i elements in the columns of the input-output matrix.

Table 5. Production matrix, with input-flows expressed in terms of total gross output

		Production	Final demand					
		Industries	Con-sumption	Gross capital formation	Exports	Total	Minus Imports	Gross output
Production	Industries	$a_{11}P_1 \quad a_{12}P_2 \ldots a_{1n}P_n$	C_1	I_1	E_1	F_1	$-M_1$	P_1
		$a_{21}P_1 \quad a_{22}P_2 \ldots a_{2n}P_n$	C_2	I_2	E_2	F_2	$-M_2$	P_2
		. . . .	. .	. .	. .	. .	. .	. .
		$a_{n1}P_1 \quad a_{n2}P_2 \ldots a_{nn}P_n$	C_n	I_n	E_n	F_n	$-M_n$	P_n
Primary inputs	Value added	$y_1 P_1 \quad y_2 P_2 \ldots y_n P_n$						
	Im-ports							
Gross input		$P_1 \quad P_2 \ldots P_n$						

18. From this table a set of input-output coefficients may be derived, which is of special importance in input-output theory. The coefficients may be presented in matrix form as follows:

$$\left.\begin{array}{l}\left.\begin{array}{cccc} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{array}\right\} \text{matrix of technical (= input-output) coefficients} \\ \begin{array}{cccc} y_1 & y_2 & \cdots & y_n \end{array} \text{ - vector of value added coefficients} \end{array}\right\} \text{(III.3) matrix of input coefficients}$$

19. The input-output coefficients define the input requirements per unit of gross output in each industry. The following input-output coefficients may be estimated from the numerical example in table 4.

.050	.097	.044	0	input-output coefficients (III.4)
.300	.437	.132	.164	
.050	.051	.059	.041	
.050	.047	.029	.137	
...	...	...	...	
.550	.368	.735	.658	- value added coefficients

20. This shows that in order to produce one unit of gross output in industry 1, .050 units of inputs produced by establishments classified in the same industry, .300 units produced by industry 2, .050 units produced by industry 3, .050 units produced by industry 4, and .550 units of value added are needed. The first assumption of input-output analysis implies that these unit requirements do not change with different scales of production. This means that the input requirements for a production of 100 or 1000 units can be found by simple multiplication of these coefficients with 100 and 1000 respectively.

21. This is the so-called simple coefficient matrix. It defines the relation between gross output of each industry and its inputs. It can serve, however, as a basis for the derivation of relations between final demand and the necessary requirements with regard to total production of each industry, value added.

This type of relation is more important, because in actual applications of input-output analysis final demand usually is a datum, while the rest of the production matrix has to be estimated.

22. The link between final demand and gross output can easily be obtained by writing down the equalities derived from the first n rows of table 5.

$$\begin{array}{l} a_{11} P_1 + a_{12} P_2 + \dots + a_{1n} P_n + (F_1 - M_1) = P_1 \\ a_{21} P_1 + a_{22} P_2 + \dots + a_{2n} P_{2n} + (F_2 - M_2) = P_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1} P_1 + a_{n2} P_2 + \dots + a_{nn} P_n + (F_n - M_n) = P_n \end{array} \qquad (III.5)$$

This can be rewritten as:

$$\begin{array}{l} (1 - a_{11}) P_1 - a_{12} P_2 - \dots - a_{1n} P_n = (F_1 - M_1) \\ - a_{21} P_1 + (1 - a_{22}) P_2 - \dots - a_{2n} P_n = (F_2 - M_2) \\ \dots\dots\dots\dots\dots\dots\dots \\ - a_{n1} P_1 - a_{n2} P_2 - \dots + (1 - a_{nn}) P_n = (F_n - M_n) \end{array} \qquad (III.6)$$

It will be clear from this set of equations (n equations and n unknown P's) that the set of P's can be expressed in terms of the set of (F-M)'s. Thus, if final demand and imports are known, gross output for each of the industries can be derived.

23. If these relations between gross output and final demand are combined with the postulated connexions between gross output and value added ($Y_i = y_i P_i$) the link between value added and final demand can be derived.

24. To compute gross output and value added per unit of final demand to be delivered by industry i, $(F_i - M_i)$ is given the unit value, i.e., $(F_i - M_i) = 1$ and all the other (F-M)'s are kept equal to 0. Applied to the example of table 4, with input coefficients as shown in (III.4), equation (III.5) can then be written in the following form:

$$
\begin{aligned}
.050\,P_1 + .097\,P_2 + .044\,P_3 + \qquad\qquad + (F_1 - M_1) &= P_1 \\
.300\,P_1 + .437\,P_2 + .132\,P_3 + .164\,P_4 + (F_2 - M_2) &= P_2 \\
.050\,P_1 + .051\,P_2 + .059\,P_3 + .041\,P_4 + (F_3 - M_3) &= P_3 \\
.050\,P_1 + .047\,P_2 + .029\,P_3 + .137\,P_4 + (F_4 - M_4) &= P_4
\end{aligned}
$$

or

$$
\begin{aligned}
(1 - .050)P_1 - .097\,P_2 - .044\,P_3 \qquad\qquad &= (F_1 - M_1) \\
- .300\,P_1 + (1 - .437)P_2 - .132\,P_3 - .164\,P_4 &= (F_2 - M_2) \\
- .050\,P_1 - .051\,P_2 + (1 - .059)P_3 - .041\,P_4 &= (F_3 - M_3) \\
- .050\,P_1 - .047\,P_2 - .029\,P_3 + (1 - .137)P_4 &= (F_4 - M_4)
\end{aligned}
$$

25. The vector of gross output (P) can be expressed in terms of the vector of final demand (F - M) by solving the equations for the P's as follows:

$$
\text{(III.7)} \quad
\begin{aligned}
P_1 &= 1.112\,(F_1 - M_1) + .182\,(F_2 - M_2) + .076\,(F_3 - M_3) + .035\,(F_4 - M_4) \\
P_2 &= .596\,(F_1 - M_1) + 1.865\,(F_2 - M_2) + .286\,(F_3 - M_3) + .349\,(F_4 - M_4) \\
P_3 &= .092\,(F_1 - M_1) + .110\,(F_2 - M_2) + 1.083\,(F_3 - M_3) + .070\,(F_4 - M_4) \\
P_4 &= .095\,(F_1 - M_1) + .109\,(F_2 - M_2) + .054\,(F_3 - M_3) + 1.179\,(F_4 - M_4)
\end{aligned}
$$

The value added equations (III.2) are in this example:

$$Y_1 = .550\ P_1$$
$$Y_2 = .368\ P_2$$
$$Y_3 = .735\ P_3$$
$$Y_4 = .658\ P_4$$

26. If these expressions are substituted in equation (III.7), the link between total value added and final demand (minus imports) is:

$$Y = Y_1 + Y_2 + Y_3 + Y_4 = 1.042\ (F_1-M_1) + 1.247\ (F_2-M_2) + .825\ (F_3-M_3) + .899\ (F_4-M_4) \quad \text{(III.8}$$

27. Gross output per unit of final demand (minus imports) to be delivered by industry 1, can be found by substituting in equations (III.7): $(F_1-M_1) = 1$ and $(F_2-M_2) = (F_3-M_3) = (F_4-M_4) = 0$. This gives $P_1 = 1.112$ $P_2 = .596$ $P_3 = .092$ $P_4 = .095$. The value added requirements for the same unit of final demand result from a substitution of the same (F-M) - values in equations (III.8). This gives Y = 1.042.

28. The same procedures can be followed in estimating the final demand components to be delivered by other industries. The overall result is presented below in matrix form, similar to the matrix shown in (III.4).

1.112	.182	.076	.035	
.596	1.865	.286	.349	cumulative input-output (= technical)
.092	.110	1.083	.070	coefficients
.095	.109	.054	1.179	(III.9)
1.042	1.247	.825	.899	- cumulative value added coefficients

This is called a matrix of cumulative input-coefficients. The first column shows that in order to produce 1 unit of final demand to be delivered by industry 1, this industry has to produce a gross output of 1.112 units, the required gross output of industry 2 is .596 units, of industry 3, .092 units and of industry 4, .095 units and the value added requirements are 1.042 units.

29. It should be noted that in this case the gross output of industry 1 is larger than the 1 unit it has to deliver to final demand. The reason is that this industry not only has to produce the final demand unit itself, but also the

raw materials, which are needed to produce this unit. And again in order to produce these raw materials, an additional amount of raw materials has to be produced by industry 1, etc. The total converges to the amount of gross output which was computed above.

30. This matrix of cumulative input coefficients is derived directly from the simple coefficient matrix. Therefore, the assumption of constant input coefficients which was applied in the case of the simple coefficient matrix is valid for the matrix of cumulative coefficients as well.

31. The assumption that the input-output coefficients are constant has two important consequences for the application of the input-output matrix. First, the assumption implies that the coefficients are valid regardless of the size of gross output or final demand. The cumulative input coefficients therefore can serve a basis for the calculation of the input requirements of any component of final demand. Secondly, the coefficients are also assumed constant over time. This makes it possible to calculate the gross output and primary inputs of each industry, which are needed to produce a given component of final demand for another year.

32. As will be shown in the next chapter, these assumptions are quite rigid and the resulting estimates might deviate considerably from actual figures. A number of techniques have been developed which allow the use of additional information concerning elements of the matrix and row and column totals, so that coefficients can be corrected for too obvious errors. The most useful of these techniques is the so-called RAS-method, developed by the Department of Applied Economics of the University of Cambridge.[1/] This method makes it possible to estimate an input-output table for a later year on the basis of the input-output coefficients for a previous year, if the row and column totals are known.

IV. RECOMMENDATIONS MADE IN THE SNA TO IMPROVE THE STABILITY OF THE INPUT-OUTPUT COEFFICIENTS

A. The limitation of the assumption of constant input-output coefficients

33. The crucial point in the applicability of the input-output theory is the actual relevancy of the three assumptions mentioned in chapter III. These assumptions grossly over-simplify reality. The deviations of the assumptions from

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1/ Input-output relations 1954-1966 in Programme for Growth, nr. 3, Department of Applied Economics, University of Cambridge, September 1963.

the actual relations between the flows in the input-output table can be subdivided into three categories.

34. First, a constant-returns-to-scale production function is assumed which implies that no economies can be gained from an increase of the production volume. This function does not take into account, as for instance the Cobb-Douglas or similar types of production functions do, that similar production processes can differ from each other in the proportion in which capital and labour are employed which affect the rest of the input structure also. Changes in productivity as a result of technological change are furthermore disregarded. The most serious deficiency of the assumption, however, is that it does not allow for the substitution of one input for another which may be the result of a completely different production process. Such substitution happens, for instance, in the building industry, where plastic materials are replacing wood for certain purposes. Substitution also often takes place between imported materials and similar domestically produced goods.

35. Secondly, deviations from the assumption may occur because the input and output flows have to be valued in money terms. The input-output assumptions are based on flows expressed in physical quantities. However, the aggregation of similar but not identical flows and the fact that some flows, such as services, cannot be expressed in quantities at all, makes it necessary to value outputs and inputs in monetary terms. Consequently, the input-output coefficients are dependent on the relative prices of inputs and outputs and any change in the relative prices affects them. Also, the prices often are not the same for each destination of a commodity. There may be large price differences, especially between the part of a flow destined for intermediate consumption and the part utilized for final demand. For instance, the government may refund commodity taxes paid, or render a subsidy, when a commodity is exported, or the quality of a product used for intermediate consumption may be inferior to the quality of the same product used for final demand and this may be reflected in the prices paid. Consequently, the average price used in valuing a flow depends on the way in which the flow is distributed on destinations.

36. Third, the fact that the outputs of an establishment usually consist of one or more subsidiary or by-products in addition to its characteristic products

violates the assumption of homogeneity of product flows and the identity of industry and product.

37. As will be shown in the following, it is possible to eliminate some of the disturbing factors mentioned above.

B. Classification of production processes

38. The input-output assumptions require that each industry should produce only one product. In practice, however, the establishment is the smallest unit which can be observed statistically, and many establishments produce several types of commodities. Where the output of uncharacteristic products such as trade of own-produced goods, repairs of such products, production of electricity for own use, etc., is small as compared with the main activity of an establishment, the stability of the input coefficients is not affected much. If, however, such activities are large or exceptional to the main activity of the industry concerned, the coefficients may be distorted to a considerable degree.

39. Considerably more effort should be made in order to distinguish the production of different commodities and classify them to their proper industry in the case of input-output tables than in the case of other aspects of national accounts estimates. If possible, only ancillary activities which are common to most establishments of an industry, minor own-account capital formation and the production of small amounts of subsidiary or by-products should be left as the uncharacteristic products of an industry.

40. In order to decide about the size of an input-output table or the number of industries to be distinguished, a few important considerations should be borne in mind. Too little detail and too much aggregation destabilizes the input coefficients. On the other hand, too much detail, makes the input coefficients too sensitive to substitution effects. This will happen, for instance, if the output of very similar products, such as different qualities of steel are distinguished although they might easily replace each other as inputs in the same production process. Also, if the statistical methods used do not make it possible to distinguish industries with a rather homogeneous output, a very large table will not be useful, because each industry will have such a heterogeneous composition that the input-output coefficients will have little meaning.

C. The treatment of trade and transport margins

41. Most of the flow of goods among industries and between industries and final demand pass through retail and/or wholesale trade. If this flow pattern was registered in the input-output table, most goods would be shown as inputs and outputs of the distribution industry and consequently most of the inputs of other industries would be delivered by that industry. This would strongly violate the assumption of homogeneity of the product flow of the distribution industry and would affect the stability of the input-output coefficients of this industry as well as all other industries which buy their inputs from trade. The obvious way to get around this difficulty is to register each flow as an output of the industry where it is originally produced and as an input of the industry which ultimately uses it. As a consequence, trade as well as transport margins should be allocated separately on uses.

42. The manner in which the trade and transport margins are allocated may also influence the stability of the input-output coefficients. The margins may be allocated either to the producer or to the purchaser of a commodity. In the first case the value of the output reflects the price which a purchaser pays for it while in the second case the value of the output excludes the margins and reflects the price which the producer actually receives.

43. The difference between valuation at purchasers' and producers' prices may be illustrated by means of an example distinguishing between (1) the industry which actually produces a commodity, (2) the industries or final demand categories which consumes them, and (3) the trade and transport sector. The producers' value is 40, the margins are 10 and the purchasers' value is 50.

Valuation at purchasers' prices

	1	2	3
1		50	
2			
3	10		
Other inputs	40		

Valuation at producers' prices

	1	2	3
1		40	
2			
3		10	
Other inputs	40		10

44. In the production matrix of table 4 valuation at producers' prices is used since deliveries by the trade and transport industry to demand categories are shown.

Valuation at producers' prices is preferable in the first place because one of the cost components which causes a different valuation for different destinations of the same industry output flow is then eliminated. Secondly, the margins are more closely linked to the output of the industry which is using the commodities produced as intermediate consumption than to the output of the industry producing them. Table 6 shows the allocation of the trade and transport margins which are shown in table 4 to the industries in which the commodities involved are characteristically produced.

Table 6. Trade and transport margins in respect of each of the flows of table 4[1/]

	Production: Industries 1	2	3	4	Final demand: Consumption	Gross capital formation	Exports	Margins classified by industries where commodities characteristically produced
Production Industries 1	0	1	0	0	1	0	0	2
2	1	10	2	8	31	0	9	61
3	0	2	0	2	7	1	2	14
4	-	-	-	-	-	-	-	-
Margins classified by user	1	13	2	10	39	1	11	77

1/ Derived from table 4.3 page 55 of the SNA.

The row totals (2, 61, 14) represent the margins classified according to the industries where the commodities in question are characteristically produced. The column totals (1, 13, 2, 10, 39 etc.) are of course the margins shown in table 4.

45. The result of valuing the output of commodities shown in table 4 at purchasers' values is given in table 7. Comparisons between table 7 and table 4 will indicate that not only are the input-output coefficients of industries 2 and 3 substantially altered, but the gross output of trade and transport margins is duplicated in the totals of the rows and columns.

Table 7. Production matrix in purchasers' values

			Production					Final demand			
			Industries				Consumption	Gross capital formation	Exports	Minus Imports	Total
			1	2	3	4					
Production	Industries	1	1	28	3		12		1	-23	22
		2	7	131	11	20	120	42	42	-35	338
		3	1	16	4	5	47	4	7	- 2	82
		4	2	61	14			1 1/		- 5	73
Primary inputs	Value added		11	102	50	48					211
	Imports										
Gross Total			22	338	82	73	179	47	50	-65	

1/ Gross fixed capital formation by the trade and transportation industries

D. The distinction between industries and commodities

46. Even with the most sophisticated statistical methods, it is not possible to define the establishment in such a way that the homogeneity assumptions of input-output analysis are completely satisfied. The result is a table of a mixed character. The columns represent industries which produce more than one product. The data on intermediate consumption are usually in product terms and are matched with the industries which produce them as their main product, while they may in fact have been produced by other industries as subsidiary or by-products.

47. This confusion between industries and their products, can be remedied by a method designed in the SNA. The method makes use of the information on the composition of outputs of establishments which usually is available from production surveys and censuses. The production process is subdivided into industries, the products of which are classified by different categories of commodities. Each industry produces main products called the characteristic products of that industry and may, in addition, produce some other products which are the characteristic products of other industries.

48. Table 8 shows how information on the commodity output of each industry may be integrated into the matrix by subdividing the production rows and columns into industry and commodity rows and columns. Each commodity category refers to the characteristic products of a particular industry.

Table 8. Production matrix for four industries and four commodities at producers' prices

			Production								Final demand			Total
			Commodities				Industries				Con-sumption	Gross capital formation	Exports	
			1	2	3	4	5	6	7	8				
Production	Commodities	1					1	27	3		11		1	43
		2					6	121	9	12	89	43	33	313
		3					1	14	4	3	40	3	5	70
		4					1	13	2	10	39	1	11	77
	Industries	5	20											20
		6		276	1									277
		7		1	67									68
		8		1		72								73
Primary Inputs	Value added						11	102	50	48				211
	Im-ports		23	35	2	5								
Gross Total			43	313	70	77	20	277	68	73	179	47	50	

49. The information on the commodity outputs of each industry is registered in the matrix formed by industry rows and commodity columns. In this "make-matrix", each row shows the commodity compositions of the output of a particular industry. The matrix formed by the commodity rows and industry columns is called an absorption matrix. In the make-matrix, the diagonal elements are the characteristic products produced by the industry itself. The size and number of off-diagonal elements shows the degree to which the establishments of a given industry are in practice homogeneous production units. These elements represent two types of commodities: subsidiary products and by-products.

50. The subsidiary products of an industry are made through different production processes than the characteristic products of the industry. Examples are electricity produced by a mining company or the construction activity of a manufacturing firm. The production of by-products and fruit products is however, inseparably linked with the production of the characteristic product of an industry. Examples of such joint products are milk and cream, coke and gas and many products of the chemical industry. Also, certain scraps and waste products, when sold, are a part of this group. However, only those products, which can be produced separately in

another type of production process appear in the non-diagonal elements of the matrix. Coke produced in gas works is an example of such a product. Milk and cream, however, are generally both produced in the dairy industry and appear as a part of its characteristic production. Scrap and waste products are also considered to be characteristic products of the industry which creates them.

51. As compared with table 4, as well as 7, final demand is in table 8 reclassified from an industry to a commodity classification. In this particular example the reclassification has little impact on the data since the amount of non-characteristic output produced by the industries is quite small and because input data of industrial production and most of the data on final demand usually are available in terms of commodities and not in terms of industries. The assumption is made that the only flow which is affected by the reclassification is the component of gross fixed capital formation which refers to increase in stocks. The vector for gross capital formation which was (0 42 3 2) in table 7 is consequently changed to (0 43 3 1) in table 8.

52. Imports are reclassified by commodity as well and placed in the intersection of commodity column and import row, eliminating the separate negative import column. Included in imports may be commodities which are not produced in the country for which the matrix is being compiled. These imports are called complementary imports: Separate categories may be introduced on the commodity columns for the complementary imports.

53. The distinction between commodities and industries provides two matrices instead of one. A method is therefore needed which can assimilate the two matrices into an input-output table such as was discussed earlier. The essence of this method and the assumptions on which it is based are explained in chapter V.

E. Basic values

54. After the distinction between industries and commodities has been made, it is easier to eliminate one of the causes of inequalities in the pricing of gross output, namely the net commodity taxes. Net commodity taxes consist of indirect taxes less subsidies, which vary with the quantities or values of commodities produced or sold. They include the taxes levied on domestic production as well as import duties. If these taxes are deducted from the value of the flows on which they are levied, the remaining value will have a more homogeneous price structure than before.

55. Each commodity flow value, which is presented in the commodity rows and columns may contain commodity taxes which are levied on its output and on the direct and indirect requirements of inputs in the successive stages of its production. Ideally, therefore, the net commodity taxes levied not only on the output of a commodity, but also those levied on its inputs and the inputs into its inputs and so on, should be deducted from the value of each commodity flow in order to completely eliminate the disturbing effect of the net commodity taxes on the input or output coefficients. The resulting value is called the "true basic value".

56. In practice, however, no information is usually available on commodity taxes levied on the indirect input requirements of each commodity flow. Because of this the SNA instead recommends the use of "approximate basic values". These are for output the producers' value minus the net commodity taxes paid by the producer and for intermediate inputs and final demand the producers' value minus the net commodity taxes paid by the last producer of the flows concerned. Approximate basic values are always larger than or equal to true basic values.

57. The production matrix in approximate basic values is shown in table 9.

Table 9. Production matrix in approximate basic values

			Production												Final demand			
			Commodities								Industries				Con-sumption	Gross capital formation	Export	Gross output
			Basic values				Commodity taxes, net											
			1	2	3	4	5	6	7	8	9	10	11	12				
Production	Basic values	1									1	19	3		12		1	36
		2									6	118	8	11	79	43	34	299
		3									1	14	2	3	36	3	5	64
		4									1	13	2	9	39	1	11	76
	Commodity taxes, net	5										8			-1			7
		6										3	1	1	10		-1	14
		7											2		4			6
		8												1				1
	Industries	9	21				-1											20
		10		267	1			9										277
		11		1	61				6									68
		12		1		71				1								73
Primary inputs	Value added			1			8	5			11	102	50	48				225
	Imports		15	29	2	5												
Gross input			36	299	64	76	7	14	6	1	20	277	68	73	179	47	50	

The commodity taxes on the output of each industry are shown in the columns "Commodity taxes, net" those on the intermediate inputs and final demand in the corresponding rows. To match with the basic price evaluation of domestic production, import flows are valued c.i.f. Import duties are included in value added, and separated between protective duties, which appear under basic values, and revenue raising duties, which appear under commodity taxes, net.

V. REDUCTION OF THE MAKE AND ABSORPTION MATRICES TO A COMMODITY X COMMODITY INPUT-OUTPUT MATRIX

58. The starting point of the input-output theory was an input-output matrix which reflected the separate input structures of commodities produced. Earlier in this paper it was explained, however, that the limitations of statistical data made observation of a pure input structure for each commodity group separately, impossible. Instead, it is only possible to observe the input structure of an industry which produces more than one commodity. The problem is how to identify, as far as is possible, the separate commodity input structures in these industry input structures.

59. In order to do this, it is desirable to decide on the pattern that the commodity input structures of the uncharacteristic products of a given industry will follow. A number of alternative decisions may be reached in respect of this question. These decisions should be based on all the engineering and technical information that can be assembled concerning the process of producing the commodities in question. In the following the consequences of one type of assumption suggested in the SNA, the commodity technology assumption is analyzed. This assumes that the same commodity, irrespective of the industry in which it is produced, has the same input structure.

60. The way the assumption is applied in reducing the make and absorption matrices into one commodity x commodity input-output matrix, may be illustrated on basis of the numerical example presented in table 9. The unknown commodity x commodity input-output matrix is presented as follows:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \qquad \text{(V.1)}$$

61. The element a_{ij} $(i, j = 1, \ldots, 4)$ is the unknown input of commodity i which is needed to produce one unit of commodity j. Take for instance, the element in row 2 and column 2 of the absorption matrix of table 9, which has a value of 118. This figure indicates that 118 basic value units of commodity group 2 are needed as input in industry 2, in order to produce 0 basic value units of commodity group 1, 267 units of commodity group 2, 1 unit of commodity group 3 and 0 unit of commodity group 4 (see the make matrix of table 9). The value of 118 is the sum of the input-values of commodity 2 into each of the commodities produced by industry 2, i.e. $0.a_{21} + 267.a_{22} + a_{23} + 0.a_{24} = 118$. The a's are the unknown commodity input-coefficients. A similar way of reasoning with regard to the other input values of the absorption matrix, results in the following set of equations (0-values are eliminated):

$$\begin{aligned}
26\,a_{11} &= 1 \\
267\,a_{12} + a_{13} &= 19 \\
a_{12} + 61\,a_{13} &= 3 \\
a_{12} + 71\,a_{14} &= 0 \\
26\,a_{21} &= 8 \\
267\,a_{22} + a_{23} &= 118 \\
a_{22} + 61\,a_{23} &= 8 \\
a_{22} + 71\,a_{24} &= 11 \qquad \text{(V.2)} \\
26\,a_{31} &= 1 \\
267\,a_{32} + a_{33} &= 14 \\
a_{32} + 61\,a_{33} &= 2 \\
a_{32} + 71\,a_{34} &= 2
\end{aligned}$$

$$\begin{aligned}
26\, a_{41} \qquad\qquad\qquad\qquad\qquad &= 1 \\
267\, a_{42} + a_{43} \qquad\qquad &= 13 \\
a_{42} + 61\, a_{43} \qquad\qquad &= 2 \\
a_{42} + \qquad\qquad + 71\, a_{44} &= 9
\end{aligned}$$

Because the number of unknown a's is equal to the number of equations, the a-values and consequently the commodity input structures are uniquely determined. The solution of (V.2) is:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} .038 & .071 & .048 & .001 \\ .308 & .445 & .124 & .149 \\ .039 & .052 & .032 & .042 \\ .039 & .049 & .032 & .126 \end{bmatrix} \qquad (V.3)$$

The value added coefficients $(y_1\ y_2\ y_3\ y_4)$ per commodity can be found in a similar way from the following set of equations:

$$\begin{aligned}
26\, y_1 \qquad\qquad\qquad\qquad &= 14 \\
267\, y_2 + y_3 \qquad\qquad &= 102 \\
y_2 + 61\, y_3 \qquad\qquad &= 50 \\
y_2 \qquad\qquad + 71\, y_4 &= 48
\end{aligned} \qquad (V.4)$$

The solution for the y's are:

$$(y_1\ y_2\ y_3\ y_4) = (.5384 \quad .3790 \quad .8135 \quad .6654). \qquad (V.5)$$

The matrix (V.3) and vector (V.5) of the input-output and value added coefficients, respectively, are comparable to the matrix of simple coefficients (III.9).

62. The commodity technology assumption is only one type of assumption which can supplement the factual information. The industry technology assumption is another one suggested in the SNA. It assumes that the input structure of all commodities produced by one industry is the same. Consequently similar commodities produced by different input structures. The commodity technology assumption is more appropriate for subsidiary products which are created by production processes separately from the characteristic production process of the industry. In the case of by-products

the industry technology assumption is more relevant, because the production process of a by-product cannot be separated from the characteristic production process. When the non-diagonal elements consist of subsidiary and by-products as well, both assumptions can be applied at the same time to different elements of the make-matrix.

63. Regardless which assumption is used to amalgamate the make and absorption matrices, the resulting input-output matrix can be handled in the same way as explained in chapter III except that outputs, intermediate inputs and final demands should be classified by commodity categories because the input-output matrix is a commodity x commodity matrix. This is an advantage because in statistical sources, uses are usually classified by commodities. In addition uses should be valued at basic values. Consequently a separate element in the intermediate and final demand vectors should be reserved for activities of the trade and transport industry, which directly relate to final demand.

64. A final remark concerns the changes of the relative prices of inputs and outputs over time. These changes disturb the relation between the input-output table and the final demand vector as well. This means that if one wishes to utilize the available information about a given period together with the input-output matrix of an earlier period, all information for the two periods should be expressed in the same prices. This means e.g., that in order to translate final demand in the given period into gross output and primary inputs per commodity in that period with the help of the input-output coefficient matrix for the earlier year, the final demand vector should be expressed in the same prices as the earlier year, to make this exercise possible.