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DIVA, A CGE Model for the study of African Diversification

Mohamed Hedi BCHIR, Hakim BEN HAMMOUDA and Mohamed Abdelbasset CHEMENGUI

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1. Introduction

Since their adoption by the Millennium Summit in September 2000, the Millennium Development Goals (MDG) have constituted the new framework of policy formulation and development strategies. The adoption of the MDGs has constituted an important advance of the international community in terms of building a consensus on development and international co-operation priorities. The adoption of this consensus has been reinforced over the last few years by the development of mechanisms aimed at monitoring progress towards achieving the MDGs. The World Summit organized by the United Nations in September 2005 provided a forum to assess the progress made by the regions of the world towards achieving these goals. In this regard, it should be noted that the majority of the regions in the world have recorded good results and have made credible progress towards achieving the goals (UNECA 2005).

However, Africa's performance has been much weaker and its progress towards achieving the MDGs was found to be lower than expectations. Certain countries, including those of North Africa and Mauritius, made important progress to reach the MDGs. But, in most of the African countries the progress was found to be relatively weak. The MDGs are the motivation of the emergence of new technical tools that aim at helping countries measure their progress in meeting the goals. Besides, new models have been built to help countries determine the policies to implement in order to sustain economic growth and achieve the MDGs. Examples of such models include the MAMS model developed by the World Bank that measures the effects of social expenditures on achieving the MDGs (Bourguigon et al. (2006) and Lofgren et al. (2006))

This work is a contribution to current efforts to build technical tools and models aimed at assisting governments to design adequate policies in order to reach MDGs. We opted for a CGE model, the most common tool used for developing countries studies, as it offers a relatively economy-wide analysis framework that takes into account interactions of complex effects necessary for MDGs realization.

This work presents a "standard" version of a CGE model dedicated to developing countries studies. This model can be applied to various economies, as it is adaptable to take into account the specificities of the economy under study.

An important contribution of this model which makes it unique in a sense that it emphasizes the role of economic growth and diversification in MDGs achievement. The issue of diversification is not recent in economic literature (Syrquin et al. (1988) and Srinivasan, on (1988)). During the 1930s crisis, Laughlin (1930) was the first to analyze the economic growth-diversification problem. He tried to explain economic cycles in American cities by the degree of concentration of economic activities. Diversification was also a preoccupation during the same period in relation to studies on the conjuncture hazard and more particularly commodities decline such as coffee in Latin America. These works on diversification developed rapidly during the 1940s and 50s and constituted the dominating paradigm in growth and development theories until the end of the 1970s. Various authors have been preoccupied by different aspects of diversification. For instance, Rosenstein-Rodan and Léontief emphasized the intersectoral matrix densification notions (W. Leontief et al., (1986), Rosenstein-Rodan et al. (1943). Empirical works on diversification have also showed that it plays an essential role in conjuncture hazard control and particularly on commodities prices fluctuations in developing countries (Massel, (1970)). On their part, Kuznets and Rostow considered structural transformation and diversification as prerequisites for growth and development (Kuznets et al. (1966) and Rostow et al. (1960)).

Diversification was at the center of pioneering works on development economics . It was at the origin of the elaboration of a series of choices in development strategy and more particularly imports substitution strategies implemented by most of the developing countries during 1960s and 70s. Meanwhile, the end of the 70s crisis and the failure of the imports substitution strategies were at the origin of a marginalization of ideas on diversification. Instead macroeconomic stabilization and international specialization became the focus of discourses on development policies. However, the last few years have witnessed a strong reemergence of the debate on diversification. From this point of view, this model attempts to take into account the recent results of the new diversification theory.

The second question at the heart of this model is poverty reduction and the Millennium Development Goals (MDGs). It is necessary to recall that the models, which were built to analyse and report progress on poverty reduction and the MDGs achievement emphasized on social expenditures, notably in the areas of education and health. This type of expenditure was earlier marginalized for a long time due to the public sector policies induced by the adoption of the Washington consensus framework that emphasized on macroeconomic stabilization and internal and external deficits reduction. These policies were accompanied by an unprecedented increase in poverty and social conditions deterioration. Today's renewed intensification of social policies in the development framework is necessary and could contribute to the improvement of social conditions. However, in this work, we also emphasize on the role of growth dynamics and economic diversification in MDGs achievement. Indeed, the renewal of the debate on diversification came with a consensus on the role of the growth dynamics. Recent literature explains the fragility of growth in the African economies and the marginalization of the continent in the world economy due to the weak diversification of its economic structures. Several authors have attempted to explain the relationship between the diversification and growth. In particular, recent works on endogenous growth emphasize the importance of the diversification. For instance, the Romer (1990) model introduced a positive externality of diversification that could be explained by the availability of inputs within the economy and can contribute to the increase of the productivity and human resources. Diversification can also contribute to economic growth by increasing the number of sectors and consequently investment opportunities and by reducing investors' risk (Acemoglu and Zilibotti (1997)). Several authors have highlighted the impact of diversification on economic growth through the stabilization of export revenues (Stanley and Bunnag (2001)).

When analyzing the diversification and growth relationships, most of the authors used econometric regression and tried to test the correlation between the two variables using various indicators for diversification. In this regard, it is necessary to mention the works of Berthélemy and Chauvin (2000) and Berthélemy and Soderling (2001) that elaborated an original methodology. At first, these studies use the traditional methodology of decomposition of the factors contributing to growth using a Cobb-Douglas production function. After computing the contribution of capital, labor and total factors productivity (TFP), they examine an econometric regression to estimate the various factors that explain the TFP. At this level, several explanatory variables are retained including the diversification level, development financing, economic openness and human resources. This methodology is interesting as it allows, through the total factors productivity, to show the contribution of diversification to the economic growth. Ben Hammouda et al. (2006) and Economic Commission for Africa (2007) recently presented evidence on the link between diversification and growth through total factor productivity for African countries.

In this model we have tried to connect the growth dynamics and the MDGs achievement to the economic diversification efforts. Besides, this model also takes into account developing countries specificities. It introduces a distinction between formal and informal sector because of the importance of the latter on developing countries economies. In addition, for the MDGs modeling, the approach used is based on the approach developed by Lofgren et al. (2006) in order to factor in the effects of the public policies.

2. Model structure:

DIVA is a recursive, dynamic multi-sectoral single country model. It distinguishes between three modes of production. The agriculture production (*AGR*) realized in rural area, the formal *(FOR)* and informal *(INF)* modes of production both realized in urban area. The three modes differ by the structure of their production, the fiscal policy that they undergo, their relationships with the financial system and the demand that they are facing. The economy consists on *N* sectors. The sectors $\{1,......,N_A\}$ are agricultural sectors realized in rural areas and the remaining $N-N_A+1$ sectors are non-agricultural and realized in urban areas. Each sector could contain formal and informal firms. For this reasons all the variables will be indexed by the couple *(i,s)* where $i\in\{1,......,N\}$ and $s\in\{AGR,FOR,INF\}$ in order to distinguish between the three modes of production within each sector.

2.1. The production block:

In each period *t* and within each sector and for each mode of production, the production function *Y(i,s,t)* is supposed to be a Leontief function that combine the Value Added *VA(i,s,t)* and the total intermediate consumption *INT(i,s,t)*. The Leontief function is defined in equation (1).

$$Y(i,mode,t)=Min\left[\frac{VA(i,s,t)}{a\_Y\_VA(i,s,t)},\frac{INT(i,s,t)}{a\_Y\_INT(i,s,t)}\right] \qquad (1)$$

Where $a\_Y\_VA(i,mode,t)$ and $a\_Y\_INT(i,mode,t)$ are the Leontiel function coefficient that can vary according to the time *t* in order to capture the effects of the variation of the production structure and technology.

Value Added

Rural Value Added

Agricultural sectors use land, capital and unskilled rural workers in order to achieve the Value Added $VA(i, AGR, t)$. The Value Added function is supposed to be a nested CES function that combines capital and land in the first stage in order to form the $K\_Land(i,t)$ composite factor that is combined with the rural unskilled rural workers $UA(i,t)$ to obtain the Value Added. This hypothesis is chosen so as to take into account the highest level of substitutability between land and capital compared to the level of substitution between land or capital and unskilled labor. Taking into account theses specificities, for every $i \in \{1,......,N_A\}$, the Value Added function for agricultural sectors is defined by equation (2).

$$VA(i, AGR, t) = A^A(t)CES(CES(Land(i,t), K(i, AGR, t)), UA(i,t)) \qquad (2)$$

$A^A(i,t)$ is a coefficient that captures the economic productivity level in agricultural sectors. This coefficient is supposed to depend on the level of public capital on agriculture and infrastructure and on the diversification index *DI(t)*[1].

Informal Value Added:

Informal production uses capital $K(i, INF, t)$ and urban informal labor $U(i, INF, t)$ as inputs. The production function is given by a one stage CES function as represented by equation (3)

$$VA(i, INF, t) = CES(K(i, INF, t), U(i, INF, t)) \qquad (3)$$

[1] The definition of $A^A(i,t)$ will be given by equation (5).

Urban Value Added

Formal production, uses Skilled labor *S(i,FOR,t)*, unskilled rural labor *U(i,FOR,t)* and capital *K(i,FOR,t)* as production factors. Using a nested two stages CES function, we try to take into account the highest substitution between skilled labor and capital. Taking into account these specificities, for every $i \in \{N_A + 1, ..., N\}$ the Value Added function for non-agricultural formal production is given by equation (4)

$$VA(i, FOR, t) = A^F(t)CES(S(i,t), K(i, FOR, t)), U(i, FOR, t)) \quad (4)$$

$A^F(i,t)$ is a coefficient that captures the economic productivity level in formal sectors. This coefficient is supposed to depend on the level of public capital on agriculture $K(G,t)$ and on the diversification index *DI(t)*. Relationships between productivity, diversification level and public capital will be presented in the next section[2].

Productivity, diversification and public policy

In DIVA, public policies are not neutral. Indeed, studies such as Barro (1990) and Kelly (1997) have already proved the contribution of public expenditure as a whole in economic growth. More recent papers have refined the analysis by distinguishing the contribution of different types of public expenditure. Fan and Rao (2003) have estimated the contribution of five types of public capital in economic growth for a panel of 43 developing countries. Their results have shown that the contribution to growth of the majority of public capital components are significantly positive, confirming the hypothesis of the non neutrality of public expenditure to growth.

In addition, and as shown by Ben Hammouda et al. (2006), economic diversification has a positive effect on economic productivity. DIVA also adopts the second element of externality. Finally and as intimated in equations (2) and (4) only agricultural and formal production modes could benefit from theses externalities. Equations (5) and (6) describe

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[2] The definition of $A^F(i,t)$ will be given by equation (6).

the relationships between productivity (approximated by coefficients $A^A(t)$ and $A^F(t)$), the level of economic diversification $DI(t)$ and the different components of public capital.

Agricultural production is supposed to benefit from diversification externalities as well as externalities coming from agricultural public capital $KG^{AGRIC}(t)$ and infrastructures public capital $KG^{INFRA}(t)$. This relationship is described by equation (6). The formal production is also supposed to benefit from diversification externalities as well as externalities coming from education public capital $KG^{EDUC}(t)$, transport and telecommunication public capital $KG^{TELEC}(t)$ and infrastructures public capital $KG^{INFRA}(t)$. This relationship is described by equation (6). The economic diversification level is defined by the normalized Hirschman index (Ben Hammouda et al. (2006)) (equation (7))

$$A^A(t) = A_0^A\left[DI(t)\right]^{\alpha_{DI}}\left[KG^{AGRIC}(t)\right]^{\alpha_{KG^{AGRIC}}}\left[KG^{INFRA}\right]^{\alpha_{KG^{INFRA}}} \qquad (5)$$

$$A^F(t) = A_0^F\left[DI(t)\right]^{\alpha_{DI}}\left[KG^{EDUC}(t)\right]^{\alpha_{KG^{EDUC}}}\left[KG^{TELEC}(t)\right]^{\alpha_{TELEC}}\left[KG^{INFRA}\right]^{\alpha_{KG^{INFRA}}} \qquad (6)$$

$$DI(t) = \frac{\sqrt[2]{SPE(t)} - \sqrt[2]{\frac{1}{N}}}{1 - \sqrt[2]{\frac{1}{N}}}, \ SPE(t) \text{ is defined as: } SPE(t) = \sum_i \left[\frac{\sum_R E^R(i,t)}{\sum_{j,R} E^R(j,t)}\right]^2 \qquad (7)$$

$E^R(i,t)$ is the level of exports of good *i* to country *R*.

Intermediate consumption

We make the hypothesis that the various modes of production follow the same shape of total intermediate consumption. As indicated by equation (9), the global demand of intermediate consumptions of sector *(j)* produced in *(s)* mode is supposed to be a CES function of various intermediate consumptions from various sectors of the economy $ICT(i,j,s,t)$.

$$INT(j,s,t)=\left[\sum_i a_{INT}(i,j,s)ICT(i,j,s,t)^{1-\frac{1}{\sigma_{INT}(j,s)}}\right]^{\frac{1}{1-\frac{1}{\sigma_{INT}(j,s)}}} \quad (9)$$

$\sigma_{INT}(j,s)$ is the elasticity of substitution between the various intermediate consumptions for the sector j and $a_{INT}(i,j,s)$ is the share of intermediate consumption of good i for the sector j.

Agricultural intermediate consumption

If the good *i* used as intermediate consumption is an agricultural good, firms have the choice between local good produced in the rural areas *IC(i,AGR,s,j,t)* and an imported good *ICImp(i,j,s,t)*. This choice is described by a CES function as described by equation (10). The imported intermediate consumption could be reached from partner R_N regions. The choice between the various origins is described by equation (11).

$$ICT(i,j,s,t)=CES(IC(i,AGR,j,s,t),ICImp(i,j,s,t)) \quad (10)$$

$$ICImp(i,j,s,t)=CES(IC^{R_1}(i,j,s,t),\cdots,IC^{R_N}(i,j,s,t)) \quad (11)$$

The non-agricultural intermediate consumption

We make the hypothesis that the intermediate consumptions of the urban products are determined by a three stages process. The first stage, presented by equation (12), consists of decomposing the non-agricultural aggregated intermediate demand on formal (*ICF(i,s,j,t))* and informal component (*IC(i,INFj,s,t))*. The second stage, presented by equation (13) serves to the dispatching of the formal demand between local component *IC(i,FOR,j,s,t)* and imported component $ICImp(i,j,s,t)$. This last supposition is known as the Armington hypothesis. Finally, the third stage, presented by equation (14), supposes that the imported products are differentiated by region of origin that allows emphasizing the importance of the business connections between the African countries and the main commercial blocks of the world $IC^{R}(i,j,s,t)$.

$$ICT(i,j,s,t) = CES(IC(i,Inf,j,s,t), ICF(i,j,s,t)) \quad (12)$$

$$ICF(i,j,s,t) = CES(IC(i,FOR,j,s,t), ICImp(i,j,s,t)) \quad (13)$$

$$ICImp(i,j,s,t) = CES(IC^{R_1}(i,j,s,t),\cdots,IC^{R_N}(i,j,s,t)) \quad (14)$$

2.1. Labor market structure and wages definition

The introduction of labor market in recent CGE models has been improved significantly with the ability to take into account the specificities of developed economies. Indeed, Agénor et al. (2003) have introduced three segmented labor markets: Agricultural labor market, unskilled labor market and skilled labor market. Bchir et al. (2005) have considered an approach similar to that of Agénor et al., but they also introduced the rural unskilled labor unemployment. Gibson (2005) does not distinguish between rural and urban labor but takes into account the distinction between formal and informal labor market. Informal sectors are supposed to be the employer of last resort. In all these studies, formal labor market is supposed to be non-competitive, wages are supposed to be fixed and resulting from collective bargaining negotiation. Their evolution according to time depends on economic variables such as unemployment rate, and economic growth. Other approaches such as Beghin et al. (1996) consider that labor market is competitive but labor mobility is not perfect. In DIVA, we opt for the first approach and consider three distinct labor markets. The first one is the unskilled agricultural labor market, the second market is the urban unskilled labor and the third one is the urban skilled labor.

Rural labor market

The rural labor market is perfectly competitive market. Labor supply is given by the rural population $L^{Rur}(t)$ supposed to grow naturally each year by $g^{Rur}(t)$. In addition an exogenous part of this population (*Mig(t))* is supposed to migrate each year to urban areas.

Having this information, the dynamics of rural population can be described by equation (15). On the other hand, agricultural labor demand results from the demand of all the agricultural sectors. The sectoral demand described by equation (16), results form the cost minimization program of each one of these sectors. Finally agricultural wage $WA(t)$ is derived from the market clearing condition (equation (17)).

$$L^{Rur}(t) = (1 + g^{Rur}(t))L^{Rur}(t-1) - Mig(t) \tag{15}$$

$$UA(i,t) = \left[\frac{\left[A^{A}(t)\right]^{\sigma_V(i,AGR)} (1 - a_V(i,AGR)) PVA(i,AGR,t)}{WA(t)} \right]^{\sigma_V(i,AGR)} VA(i,AGR,t) \tag{16}$$

$$L^{Rur}(t) = \sum_i UA(i,mode,t) \tag{17}$$

Unskilled urban labor market

The supply of unskilled labor market corresponds to the unskilled urban population ($L^{Urb}(t)$). This population grows each year by a natural growth rate $g^{Urb}(t)$ and by the level of migrants that come from rural area $Mig(t)$. In addition, an exogenous part $Skl(t)$ of the population is supposed to become skilled and offers its work force in the skilled labor market. Having this description, the urban unskilled labor force evolution is given by equation (19). On the other hand, DIVA considers two types of unskilled rural labor demand. The first one emanates from the formal firms. Workers in formal firms are paid at a fixed wage $W_M(t)$ that grows each year by $g^{W_U(t)}$ (equation (19)). This growth rate is supposed to results from social negotiations and takes into account the rate of inflation ($g^{CPI(t)}$) and the evolution of the wages in the informal sectors ($g^{W_U(t)}$)[3]. $g^{W_M}(t)$ is supposed to be a Cobb-Douglass function of $g^{W_U(t)}$ and $g^{CPI(t)}$ (equation (20)).

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[3] This variable is taken as a proxy of the unskilled workers unemployment rate

$$L^{Urb}(t) = (1 + g^{Urb}(t))L^{Urb}(t-1) + Mig(t) - Skl(t) \quad (18)$$

$$W_M(t) = (1 + g^{W_M}(t))W_M(t-1) \quad (19)$$

$$g^{W_M}(t) = CD(g^{CPI(t)}, g^{W_U(t)}) \quad (20)$$

The second type of unskilled rural labor demand emanates from the informal firms. In DIVA we made the hypothesis that the surplus of unskilled urban labor $UT^I(t)$ is totally absorbed by informal sectors. Thus the offer of labor addressed to informal sectors is given by equation (21). The labor demand by informal market consists of the sum of the labor demand of all the informal firms. The sector-based demand results from conditions of cost minimization program (equation (22)). The wage level in the informal labor market $W_{\inf}(t)$ results from the market clearing condition for informal unskilled urban labor (equation (23))

$$UT^I(t) = L^{Urb}(t) - UT^F(t) \quad (21)$$

$$U(i, INF, t) = \left[\frac{(1 - a_{VA}(i, INF))PVA(i, INF, t)}{W_{Inf}(t)}\right]^{\sigma_{VA}(i,INF)} VA(i, INF, t) \quad (22)$$

$$UT^I(t) = \sum_i U(i, INF, t) \quad (23)$$

Skilled labor market

In each period, the number of skilled workers $L^{Skl}(t)$ available in the economy corresponds to its level in previous year to which we add the new qualified workers that arrive on the labor market. The dynamic of the skilled labor supply is then given by equation (24). The skilled workers wage is supposed to grow every year by $g^{W_S}(t)$ (equation (26)). $g^{W_S}(t)$ depends on the inflation rate $g^{CPI}(t)$ and on unemployment rate $g^{Unp}(t)$. The relationship between $g^{W_S}(t)$, $g^{CPI}(t)$ and $g^{Unp}(t)$ is given by equation (25).

$$L^{Skl}(t) = L^{Skl}(t-1) + Skl(t) \tag{24}$$

$$g^{W_S}(t) = \alpha g^{CPI}(t) - \beta g^{Unp}(t) \tag{25}$$

$$W_S(t) = (1 + g^{W_S}(t))W_S(t-1) \tag{26}$$

For this level of salary, $W_S(t)$, the number of qualified workers engaged by formal firms, *ST(t)*, is the sum of all the demands skilled labor that emanates from formal firms *S (i, t)*. The rest of the qualified population remains unemployed. The cost minimization program of each formal firm gives the skilled labor demand *S (i, t)*, presented by equation (27) and (28).

$$S(i,t) = \left[\frac{a_{K\_S}(i)PK\_S(i,t)}{W_S(t)}\right]^{\sigma_{K\_S}} K\_S(i,t) \tag{27}$$

$$K\_S(i,t) = \left[\frac{\left[A^F(t)\right]^{\sigma_V(i,FOR)} a_V(i,FOR)PVA(i,FOR,t)}{PK\_S(i,t)}\right]^{\sigma_V(i,FOR)} VA(i,FOR,t) \tag{28}$$

2.2. Households demand block

The consumption demand of households follows the same structure as the intermediate consumption. The households make at first the choice between the consumption of various products *CT(i,t)*. They can then choose between the product produced in agricultural, formal and informal modes. For the case of formal products, they will make an Armington bargaining between local and imported products. Finally, for imported products they will choose between products from the diverse commercial partners. The welfare function $U(t)$ adopted in DIVA is an LES-CES one (equation (29)):

$$U(t) = \left[\sum_i a_U(i)(CT(i,t) - C_{Min}(i))^{1-\frac{1}{\sigma_U}} \right]^{\frac{1}{1-\frac{1}{\sigma_U}}} \quad (29)$$

$C_{Min}(i)$ is the subsistence consumption of good i, σ_U s, is the elasticity of substitution between the various final consumptions and $a_U(i)$ is the share of consumption of good i in the total consumption.

Agricultural final consumption:

For the agricultural products, the household consumption is modeled by a two levels nested CES function. In the first level, the households choose between the local good produced in rural environment *C(i,AGR,s,j,t)* and the imported good *CImp(i,j,s,t)*, while in the second level, they choose products according to their regions $C^R(i,t)$. This structure is represented by the equations (30) and (31).

$$CT(i,t)=CES(C(i,AGR,t),CImp(i,t)) \quad (30)$$

$$CImp(i,t) = CES(C^{R_1}(i,t),\cdots,C^{R_N}(i,t)) \quad (31)$$

Non-agricultural final consumption:

Final demand of non-agricultural products is modeled by a system of three levels nested CES function. At the first level, the households choose between the formal and informal products (equation (32)). In the second level, they decide between non-agricultural imported non-agricultural domestic products and non-agricultural imported products (equation (33)). Finally, the third level models the origin of the imports of non-agricultural products according to the various regions of the world (equation (34)).

$$CT(i,t) = CES(C(i,INF,t), CF(i,t)) \quad (32)$$

$$CF(i,t) = CES(C(i,FOR,t), CImp(i,t)) \quad (33)$$

$$CImp(i,t) = CES(IC^{R_1}(i,t), \cdots, IC^{R_N}(i,t)) \quad (34)$$

C(i,INF,t) represents the final consumption of good *i* produced in an informal way and *CF(i,t)* is the final consumption of good *i* produced in a formal way. *C(i,FOR,t)* is the local final formal consumption and *CImp(i,t)* is the imported final consumption.

2.3. Government demand block

The government is supposed to have two types of spending. Current spending *G(t)* and investment expenditure. The public demand of final goods follows the same structure as the households consumer structure with the exception that the government is supposed not to ask for informal products. The demand of the government of final products is deducted from a decision of costs minimization under the hypothesis a CES objective function (equation (35)).

$$\underset{i}{Min} \quad PG(t)G(t) = PCGT(i,t)CGT(i,t)$$

$$Sc/ \quad G(t) = \left[\sum_i a_G(i)(CGT(i,t))^{1-\frac{1}{\sigma_G}} \right]^{\frac{1}{1-\frac{1}{\sigma_G}}} \quad (35)$$

CGT(i,t) being the final consumption of the government of good *i* in the period *t*. It is afterward assigned between the various modes of production according to the same mechanism described for the final consumption of the household.

Agricultural public consumption:

For the agricultural goods, the modeling of the government demand is similar to that of the households. At first, the government chooses between the local products *CG(i,AGR,s,j,t)* and imported products *CGImp(i,j,s,t)* (equation (36)). In the second level, the choice of the government imported products based on the origin of imported agricultural products (equation (37)).

$$CGT(i,t)=CES(CG(i,AGR,t),CGImp(i,t)) \quad (36)$$

$$CGImp(i,t) = CES(CG^{R_1}(i,t),...,CG^{R_N}(i,t)) \quad (37)$$

Non-Agricultural public consumption:

For the non-agricultural products, the government demand is limited to formal products. The demand is deducted from a two level-nested CES system. In the first level, the government chooses between the imported and domestic formal products (equation (38)). In the second level, a distinction between the origins of the imported products is considered (equation (39))

$$CGF(i,t) = CES(CG(i,FOR,t),CGImp(i,t)) \quad (38)$$

$$CGImp(i,t) = CES(CG^{R_1}(i,t),...,CG^{R_N}(i,t)) \quad (39)$$

CG(i,FOR,t) represent the local final public consumption and *CGImp(i,t)* the imported public consumption.

2.5. International trade:

In most single countries CGE model, firms have the choice between internal and external market. Firms make their choice according to the relative prices and maximizing their profit constrained by a CET (*Constant Elasticity of Substitution*) function and suppose that all the exports offer is absorbed by foreign market. This last supposition seems not to be very accurate as many developing countries face a problem when selling their products abroad because they do not take into consideration the importing countries demand. To avoid this problem, DIVA is innovative. The innovation consists of linking DIVA to a global model that can generate demand and world price vectors. These vectors from the global model are then plugged into the DIVA model. This way, the exports and the world prices are considered as exogenous in DIVA. The total imports $IMP^R(i,t)$ of good i from a region R is defined as the sum of the demand of imported goods of the different agents of the economy: The households, the government, intermediate consumption and capital good. This definition can be written as:

$$IMP^R(i,t) = C^R(i,t) + \sum_{j,s} IC^R(i,j,s,t) + \sum_{j,mode} KG^R(i,j,s,t) + CG^R(i,t) \quad (40)$$

2.6. Investment

Within every sector, the model considers two types of investments: the public investment and the private investment. The first one is supposed to be exogenous and depends on government choices and priorities. The second is supposed to be endogenous and depends on the profitability of the sector, the degree of diversification of the economy and the level of public investment

Private investment

In CGE models, private investment is introduced in various manners. Agénor et al. (2003) developed an investment function that depends on the stock of initial capital, the evolution of the public infrastructure, the evolution of the GDP, the level of inflation, the national interest rate, the interest rate on the national debt, and the ratio of the national debt. Gibson (2006) introduced a ratio of investment on capital that depends linearly on initial investment level, the ratio of production capacity on production, the profit rate or the interest rate, public investment GDP rate and on inflation.

In this model, we distinguish three forms of investment namely the agricultural investment, the formal and informal urban investment. The public investment is considered as exogenous. Besides, we make the hypothesis that investment is explained by the following variables: level of initial capital $K(i,s,t-1)$, net return on the capital $RK(i,s,t)$, domestic interest rate $IR(t)$, inflation rate $CPI(t)$, ratio of the public investments $INVPUB(i,s,t)$ on the GDP and the indication of diversification $DI(t)$. Equation (41) determines the rural investment and equation (42) determines the formal urban investment. The informal investment is supposed to be equal to the informal sectors saving.

$$\frac{INV(i,AGR,t)}{K(i,AGR,t-1)} = f\left(IR(t), RK(i,AGR,t), CPI(t), \frac{INVPUB(i,AGR,t)}{GDP(t)}, DI(t)\right) \tag{41}$$

$$\frac{INV(i,FOR,t)}{K(i,FOR,t-1)} = f\left(IR(t), RK(i,FOR,t), CPI(t), \frac{INVPUB(i,FOR,t)}{GDP(t)}, DI(t)\right) \tag{42}$$

Public investment

The sectoral public investment $INVPUB(i,s,t)$ is supposed to be exogenous and realized only in agricultural and formal urban sectors. Its value is added up to the private investment in order to obtain the total sectoral investment $INVTOT(i,s,t)$. Equation (43) defines the total sectoral investment.

$$INVTOT(i,s,t) = INV(i,s,t) + INVPUB(i,s,t) \tag{43}$$

Capital good demand

The investment decision is going to bring firms to get capital goods that by hypothesis are produced only by the formal companies in this model. The producer's barganning between various capital goods *KGT(i,j,s,t)* will take the shape of a CES function (equation (44)). For the capital good equipment, the producers will have the choice between those locally produced (*KGLoc*) and those imported (*KGImp*) (equation (45)). For the imported capital goods, the producers will choose between the diverse exporters as described by equation (46).

$$INVTOT(j,s,t) = \left[\sum_i a_{INV}(i,j,s)(KGT(i,j,s,t))^{1-\frac{1}{\sigma_{INV}(j,s)}} \right]^{\frac{1}{1-\frac{1}{\sigma_{INV}(j,s)}}} \quad (44)$$

$$KG(i,j,s,t) = CES(KG(i,FOR,j,s,t), KGImp(i,j,s,t)) \quad (45)$$

$$KGImp(i,j,s,t) = CES(KG^{R_1}(i,j,s,t),...,KG^{R_N}(i,j,s,t)) \quad (46)$$

2.4. Prices definitions

Production prices

Production prices are deduced assuming perfect competition in all sectors and for all production modes, the zero profit condition (equation (47)). The Value Added prices are determined as a function of the volume and the process of the factors used by each sector and each mode of production. For agricultural sectors, this relationship could be represented by equations (48) and (49). Equation (50) represents the informal sectors while equations (51) and (52) represent the formal sectors:

$$PY(i,s,t) = \frac{PVA(i,s,t)VA(i,s,t) + PINT(i,s,t)INT(i,s,t)}{Y(i,s,t)} \quad (47)$$

$$PVA(i,AGR,t) = \frac{PK\_Land(i,t)K\_Land(i,t) + W_A(t)UA(i,t)}{VA(i,AGR,t)} \quad (48)$$

$$PK\_Land(i,t) = \frac{PK(i,AGR,t)K(i,AGR,t) + PLand(i,t)Land(i,t)}{K\_Land(i,t)} \quad (49)$$

$$PVA(i,INF,t) = \frac{PK(i,INF,t)K(i,INF,t) + W_{Inf}(t)U(i,INF,t)}{VA(i,INF,t)} \quad (50)$$

$$PVA(i,FOR,t)VA(i,FOR,t) = PK\_S(i,t)K\_S(i,t) + W_M(t)U(i,FOR,t) \quad (51)$$

$$PK\_S(i,t) = \frac{PK(i,FOR,t)K(i,FOR,t) + W_S(t)S(i,t)}{K\_S(i,t)} \quad (52)$$

The total intermediate consumption price is given by equation (53) while the price of agricultural intermediate consumption is given by equation (54). Equation (55) gives the prices of imported agricultural intermediate consumption. Finally, following the example of the agricultural intermediate consumption prices, non-agricultural intermediate consumption prices are determined by the equations (56, 57 and 58).

$$PINT(j,s,t) = \frac{\sum_i PICT(i,j,s,t)ICT(i,j,s,t)}{INT(i,s,t)} \quad (53)$$

$$PICT(i,j,s,t) = \frac{PIC(i,AGR,t)IC(i,AGR,j,s,t) + PICImp(i,j,s,t)ICImp(i,j,s,t)}{ICT(i,j,s,t)} \quad (54)$$

$$PICImp(i,j,s,t)ICImp(i,j,s,t) = \sum_R PIC^R(i,t)IC^R(i,j,s,t) \quad (55)$$

$$PICT(i,j,s,t) = \frac{PIC(i,INF,t)IC(i,INF,j,s,t) + PICF(i,j,s,t)ICF(i,j,s,t)}{ICT(i,j,s,t)} \quad (56)$$

$$PICF(i,j,s,t) = \frac{PIC(i,FOR,t)IC(i,FOR,j,s,t) + PICImp(i,j,s,t)ICImp(i,j,s,t)}{ICF(i,j,s,t)} \quad (57)$$

$$PICImp(i,j,s,t) = \frac{\sum_R PIC^R(i,t)IC^R(i,j,s,t)}{ICImp(i,j,s,t)} \quad (58)$$

Market prices

The linkage between production prices and market prices is dealt with when considering indirect taxes (equation (59)).

$$PC(i,s,t) = (1+\tau_c(i,s,t))PY(i,s,t) \quad (59)$$

Where $\tau_c(i,mode,t)$ is the tax rate applied on final consumption of good *i*. The tax rate is supposed to vary according to the sector and the production mode. Intermediate consumptions, capital goods and public consumptions are supposed not to be subject to direct taxation. Thus their market prices are equal to their production prices. Tariffs rates are differentiated by products, mode of production and type of use (final consumption, intermediate consumption, capital good or public consumptions). Tariff rates also vary by partners.

Taking into account all these specifications, for imported products from regions *R* and used as final consumption, equation (60) determines their prices. On the other hand, equation (61) determines the prices of products imported as intermediate consumption. The price of the imported capital goods, is given by equation (62). For the public consumption, the model assumes that no tax is applied. Thus, the prices of local goods are equal to the production prices (equation (63)) and the prices of imported products are equals to the FOB prices (equation (64)). Finally, the hypothesis of the small country retained in DIVA implies that the world prices are exogenous. Equation (65) determines the level export prices in local currency.

$$PC^{R}(i,t) = ER(t)(1+\tau_{\mathrm{Im}\,p}(i,R,t))(1+\tau_{c}(i,t))wpe(i,t) \tag{60}$$

$$PIC^{R}(i,t) = ER(t)(1+\tau_{\mathrm{Im}\,p}(i,R,t))wpe(i,t) \tag{61}$$

$$PKG^{R}(i,t) = ER(t)(1+\tau_{\mathrm{Im}\,p}(i,R,t))wpe(i,t) \tag{62}$$

$$PCG(i,s,t) = PY(i,s,t) \tag{63}$$

$$PCG^{R}(i,t) = ER(t)wpe(i,t) \tag{64}$$

$$PE(i,t) = ER(t)Wpe(i,t) \tag{65}$$

Good and services market equilibrium

Equation (66) determines the equilibrium condition on informal market where the demand is limited to the local final consumption and to local intermediate consumption. Equation (67) describes the agricultural market, it takes into account the foreign trade. Equation (68) describes formal market, total demand is constituted by the final consumption, intermediate consumption, capital good and the external demand. The total offer is constituted by the local production and exports.

$$Y(i,INF,t) = C(i,INF,t) + \sum_{j,s} IC(i,INF,j,s,t) \tag{66}$$

$$Y(i,ARG,t)+\sum_{R}C^{R}(i,ARG,t)+\sum_{R}CG^{R}(i,ARG,t)+\sum_{r,j,s}IC^{R}(i,j,ARG,t)$$
$$=C(i,ARG,t)+CG(i,ARG,t)+\sum_{j,s}IC(i,ARG,j,s)+\sum_{R}E^{R}(i,t) \quad (67)$$

$$Y(i,FOR,t)+\sum_{R}C^{R}(i,FOR,t)+\sum_{R}CG^{R}(i,FOR,t)\sum_{r,j,s}IC^{R}(i,j,FOR,t)+\sum_{R,j,s}KG^{R}(i,j,s,t)$$
$$=C(i,FOR,t)+CG(i,FOR,t)+\sum_{j,s}IC(i,FOR,j,s)+\sum_{j,s}KG(i,FOR,j,s,t)+\sum_{R}E^{R}(i,t) \quad (68)$$

2.8. Profits and Incomes.

Brut profits:

As in most CGE models, the profit formation in DIVA is defined by a simple accounting structure. The brut profit of a firm *(i,s)* is noted $PROF(i,s,t)$ and is defined as the difference between the Value Added and the labor cost. Equations (69), (70) and (71) give the structure of brut profit according to the mode of production. Agriculture sectors use only rural production, informal sectors use only urban unskilled labor and formal sectors use both skilled an unskilled urban labors force.

$$PROF(i,AGR,t)=PVA(i,AGR,t)VA(i,AGR,t)-W_{A}(t)UA(i,t) \quad (69)$$

$$PROF(i,INF,t)=PVA(i,INF,t)-W_{INF}(t)U(i,INF,t) \quad (70)$$

$$PROF(i,FOR,t)=PVA(i,FOR,t)VA(i,FOR,t)-W_{S}(t)S(i,t)-W_{M}(t)U(i,FOR,t) \quad (71)$$

Net profit

To define the net profit of a firm *(i,s)*, we take into account the payment of the interest on previous years loans. Firms are supposed to have access to domestic loans

$DL(i,s,Bank,t-1)$ from domestic private Banks and foreign loans $FL(i,s,t-1)$ from foreign banks. The agricultural sectors are supposed to have access only to domestic loans. Informal sectors are supposed to have no access to financial system and formal sectors are supposed to have access to both domestic and foreign loans. Having this three constraints, the net profit $NPROF(i,s,t)$ is then defined according to the type of activities by equations (72), (73) and (74).

$$NPROF(i,AGR,t) = PROF(i,AGR,t) - IL(t-1)DL(i,AGR,t-1) \quad (72)$$

$$NPROF(i,INF,t) = PROF(i,INF,t) \quad (73)$$

$$NPROF(i,FOR,t) = PROF(i,FOR,t) - IL(t-1)DL(i,FOR,t-1) - IF(t-1)FL(i,FOR,t-1)ER(t) \quad (74)$$

$IL(t-1)$ is the interest rate defined by domestic banks at *t-1* and *IF(i,t-1)* is the interest rate applied by foreign banks. The profit of banks corresponds to the difference between the interests that they perceive and which form their incomes and the interests that they pay on the households' deposits and on the loans that they contract from abroad. Equation (75) gives the definition of banks profit $POFPB(t)$.

$$POFPB(t) = IL(t-1)\left[\sum_{i,s} DL(i,s,t-1) + DLG(t-1)\right] - ID(t)DD(t-1) - IF(t)ER(t)FLB(t) \quad (75)$$

DLG(t) is the level of government loans from domestic banks and *FLB(t)* is the loans of commercial banks from abroad.

Revenues:

Household's revenues

Households' revenue has three sources: Labor income, banks profits that are shared in their totality[4] to households and the distributed part of firm's profits. The distributed part of firms profit (*DPROF(i,s,t)*) is described by equation (76).

$$DPROF(i,s,t) = (1-\tau_{SAV}^{f}(i,s,t))(1-\tau_{f}(i,s,t))NPROF(i,s,t) \qquad (76)$$

$\tau_{SAV}^{f}(i,s,t)$: is the rate of firms self-financing and $\tau_{f}(i,s,t)$ is the direct income tax rate on firms' profits. This rate is supposed to vary according to sectors and production mode. Thus, the before taxation revenue of households *YH(t)* is defined as the sum of the three revenue sources and given by equation (77).

$$YH(t) = W_S(t)\sum_i S(i,t) + W_M(t)\sum_i U(i,FOR,t) + W_{Inf}(t)\sum_i U(i,INF,t) + W_A(t)\sum_i UA(i,t) + PROFPB(t) + \sum_{i,s} DPROF(i,s,t) + ID(t)DD(t-1) + TROW(t)ER(t) \qquad (77)$$

ID(t): is the interest rate paid by the local banks on households deposit *DD(t)* and *TROW(t)* is the level of foreign transfers to households (remittances). Having these definitions, the net households revenue *YD(t)* is defined by equation (78).

[4] In several developing countries, a part of banking system is state owned. For this reason, a part of banks profits versed directly to the government. In DIVA and for a simplification of the writing we consider this transfer as a part of the taxes which the banking sector is paying.

$$YD(t) = (1 - \tau_H(t))YH(t) \qquad (78)$$

Where $\tau_H(t)$ is the rate of direct taxation on households' revenue. Households are supposed to save $\tau^H_{SAV}(t)$ part of their revenue and to allocate the rest to consumption. The households' budget allocated to consumption $BUGC(t)$ is defined in equation (79) and the level of households saving $SAV(t)$ is defined in equation (80). Finally, the saving rate is supposed to vary according to the interest rate and inflation rate (equation (81)).

$$BUGC(t) = (1 - \tau^H_{SAV}(t))YD(t) \qquad (79)$$

$$SAV(t) = \tau^H_{SAV}(t)YD(t) \qquad (80)$$

$$\tau^H_{SAV}(t) = f(ID(t), CPI(t)) \qquad (81)$$

2.9. Financial sector:

The model takes into account two financial institutions : The Central Bank and commercial banks. All the economic actors are supposed to interact in the financial market:

Households

The households' wealth level $(WT^H(t))$ evolution is defined by equation (82), it is equal to the sum of its level in last period and the current saving. Households are supposed to hold their wealth in two forms: a monetary form *H(t)* and a domestic deposit *DD(t)*[5] (equation (83)). The model considers that the level of the monetary form of wealth depends on the inflation rate, households' income and the level of interest rate. $H(t)$ is as described by equation (84). In addition, monetary income is supposed to be adapted to the demand side.

[5] Here we suppose that households have no access to external financial system.

$$WT^H(t) = WT^H(t-1) + SAV^H(t) \quad (82)$$

$$WT^H(t) = H(t) + DD(t) \quad (83)$$

$$H(t) = CPI(t)\left[\frac{YD(t)}{CPI(t)}\right]^{\sigma_H} (1 + ID(t))^{-\beta_D} (1 + g^{CPI}(t))^{-\beta_{CPI}} \quad (84)$$

Firms:

Firms finance their investments plans by self-financing, local banking credits and foreign banking credits. The equilibrium condition between finance demand and supply is given by equation (85).

$$\begin{aligned} PINV(i,s,t)INV(i,s,t) &= s_{SAV}^{f}(i,s,t)(1-\tau_f(i,s,t))NPROF(i,s,t) + \\ &[DL(i,s,t) - DL(i,s,t-1)] + ER(t)[FL(i,s,t) - FL(i,s,t-1)] \end{aligned} \quad (85)$$

Commercial Banks:

Commercial banks in the model are required to keep a portion $0 < rreq(t) < 1$ of the deposits that they collect as reserve requirements. The total amount of the requested deposit (*RR(t))* is then given by equation (86). Furthermore, commercial banks have to face the financing needs of private firms and the government. The model assumes that they cover all these needs using household deposit and foreign loans. The demand for foreign loans from commercial banks $FLB(mode,t)$ is by extension given by equation (87). The difference between loans interest rate and deposit interest rate forms the commercial banks margin. $\eta(t)$ the spread between loan interest and deposit interest is given by equation (88).

$$RR(t) = rreq(t)DD(t) \tag{86}$$

$$ER(t)(FLB(t) - FLB(t-1) = \left[\sum_{i,s} DL(i,s,t) - \sum_{i,s} DL(i,s,t-1)\right] + \left[DLG(t) - DLG(t-1)\right] - (1 - rreq(t))\left[DD(t) - DD(t-1)\right] \tag{87}$$

$$IL(t) = ID(t) + \eta(t) \tag{88}$$

Central bank and current account balance

In the model, Central Bank has four functions: monetary emission (*MB(t)*), reserve requirements collection from commercial banks, domestic credit to the government (*DCG(t)*) and the management of foreign currency reserves (*FF(t)*). Monetary emission dynamics is governed by equation (89). The equilibrium condition between money demand and money supply is given by equation (90). The central bank profit is defined by equation (91).

$$MB(t) - MB(t-1) = \left[DCG(t) - DCG(t-1)\right] + ER(t)\left[FF(t) - FF(t-1)\right] \tag{89}$$

$$H(t) = MB(t) - RR(t) \tag{90}$$

$$PROFCB(t) = IL(t-1)DCG(t) + IFG(t)FF(t-1) \tag{91}$$

Where *IFG(t)* is the central bank interest rate applied on domestic credit to the government. Giving these equilibrium conditions, the Central Bank foreign currency reserves (*FF(t)*) evolution is given by equation (92).

$$FF(t) - FF(t-1) = IFG(t)FF(t-1) + \sum_{i,R}\left[wpe(i,t)E^{R}(i,t) - wpm(i,t)M^{R}(i,t)\right] -$$
$$IF(t-1)\sum_{i,s}FL(i,s,t-1) - IF(t-1)FLB(t-1) - IFG(t-1)FLG(t-1) +$$
$$\left(FLG(t) - FLG(t-1)\right) + \left[\sum_{i,s}FL(i,s,t) - FL(i,s,t-1)\right] + \left(FLB(t) - FLB(t-1)\right) + TROW(t) \qquad (92)$$

2.10. Public sectors:

The model treats the public deficit question by modeling separately the public spending and the government incomes. The government is supposed to have two types of expenditure. The first type is the investment $INVPUB(i,t)$ and the second type is the current expenditure *G(t)*. The total expenditure $EXPG(t)$ is then given by equation (93). The Government revenue is defined as the sum of indirect taxes (tariffs and consumption taxes) and direct taxes (taxes on firms profit and households income). The Government revenue $REVG(t)$ is then given by equation (94). The public deficit is then defined as the difference between current revenue and the current expenditure and the payment of interest on local and foreign deficit. The government deficit $DEF(t)$ is then defined by equation (95) . This deficit is financed, as indicated by equation (96), by credits from private banks $\left(DLG(Banks,t) - DLG(Banks,t-1)\right)$, central bank $(DCG(t-1) + DLG(t-1))$ and from abroad $\left(FLG(t) - FLG(t-1)\right)$.

$$EXPG(t) = \sum_{i,s}PINV(i,s,t)INVPUB(i,s,t) + PG(t)G(t) \qquad (93)$$

$$REVG(t) =$$
$$\tau_H(t)YH(t) + \sum_{i,s}\tau_f(i,s,t)NPROF(i,s,t) + PROFCB(t) + \sum_{i,s}\tau_c(i,t)\big(PY(i,s,t)C(i,s,t)\big) +$$
$$ER(t)\sum_{R,i}\big((1+\tau_{\mathrm{Im}\,p}(i,R,t))(1+\tau_c(i,t)) - 1\big)wpe(i,t)C^R(i,t) +$$
$$ER(t)\sum_{R,i}\tau_{\mathrm{Im}\,p}(i,R,t)wpe(i,t)\big((IC^R(i,t) + KG^R(i,t)\big)$$
(94)

$$-DEF(t) = REVG(t) - EXPG(t) - IFG(t)ER(t)FLG(t-1) - \big(DCG(t-1) + DLG(t-1)\big)$$
(95)

$$DEF(t) = ER(t)\big(FLG(t) - FLG(t-1)\big) +$$
$$((DLG(t) - DLG(t-1)) - IL(t-1)(DCG(t-1) + DLG(t-1))$$
(96)

2.11. The MDGs block

The MDG block is another area where this model makes significant contribution to the ongoing debate on policies to be implemented to achieve the MDGs. In this model we try to introduce five objectives among those defined by the Millennium Declaration. These include those related to poverty reduction; access to primary education; reduction of the infant mortality for the under five years old; and improvement of the maternal health. Finally, the model introduces objective seven related to increasing sustainable access to water purification services.

Poverty reduction

CGE models constitute one of the best tools of poverty and incomes distribution analysis. Broadly speaking, the works on poverty are often criticized for the choice of poverty or disparity indexes or for the choice of the poverty line. In general equilibrium framework, we do not try to measure poverty indicators but rather the effects of the economic reforms or external shocks on these indicators variation. Indeed, simulations are based on well-established estimations of poverty indicators in order to connect economic policies simulations to the realization of the first objective of the Millennium Development Goals.

Thus, it is indispensable to have estimation of the new number of poor associated with every simulation and for every period. Considering this objective, two alternative approaches could be used to measure the effect of reforms of the economic policies on the number of poor: the micro-simulation approach and the elasticity growth of the reduction of the poverty approach. Appendix 3 describes in a more technical way these two approaches.

Access to primary education

The educational system consists of *C* cycles, within every cycle *c* students can have *B* possible behavior: repeating; (*rep)*, abandonment *(dropout*, success *(grd)*, obtaining diploma and continuing studies in the following cycle *(grdcont)*, obtaining diploma and abandoning studies in the following cycle *(grdexit)*, entry to the first year of primary studies *(g1entry*), obtaining a diploma for the last cycle of the educational system (*grdcyc)* and the success within a cycle without obtaining a diploma *(contcyc)*.

The behavior governed by a Logit function:

Three behaviors are supposed to be governed by a Logit functions: the passage of a class to the following one within the same cycle (*grd)*, the passage from a cycle to the following cycle (*grdcont)* and the entry to the first cycle of study, *(g1entry)*. The number of the students having a behavior *b*, in the cycle *c* in the period *t* is noted $SHed(b,c,t)$ and is supposed to be governed by equation (98).

$$SHed(b,c,t) = exted(b,c) + \frac{\alpha_{ed}(b,c)}{1+\gamma_{ed}(b,c)\exp(\beta_{ed}(b,c)\left(SHed\,\mathrm{int}(b,c,t) - SHed(b,c,t_0)\right)} \quad (98)$$

Where $exted(b,c)$ is the maximal number of students with the behavior *b* in the cycle *c* observed in the world. $SHed(b,c,t_0)$ is the maximal number of students with the behavior *b* in the cycle *c* at the base year, $\alpha_{ed}(b,c), \beta_{ed}(b,c)$ and $\gamma_{ed}(b,c)$ are calibrated parameters and $SHed\,\mathrm{int}(b,c,t)$ is an intermediate variable defined by equation (99) that depends on the quality of the education $EDQUAL(c,t)$. As defined by equation (100), the education quality is approximated by the evolution of the ratio public spending by educational cycle $QG(c,t)$ by the number of students in this cycle $N(c,t)$.

$$SHed\,\text{int}(b,c,t) = SHed(b,c,t_0)\left[EDQUAL(c,t)\right]^{\varphi_{EDQUAL}} \left[\frac{W(c_2,t)}{W(c_1,t)}\right]^{\varphi_{wage\_prem1}} \left[\frac{W(c_3,t)}{W(c_2,t)}\right]^{\varphi_{wage\_prem2}} \left[CFH(t)\right]^{\varphi_{C^H}} \left[MDG\_VAL(MDG_4,t)\right]^{\varphi_{MDG_4}} \left[\sum_{i\in INS} EDUC\_INF(i,t)\right]^{\varphi_{EDUC\_INF}} \tag{99}$$

$$EDQUAL(c,t) = \frac{QG(c,t)}{N(c,t)}\left[\frac{QG(c,t_0)}{N(c,t_0)}\right]^{-1} \tag{100}$$

$W(c_1,t)$, $W(c_2,t)$ and $W(c_3,t)$ being level of the salaries of the individual who leave the educational system during respectively the primary, secondary and superior cycle. The value of the fourth MDG in the period t will be defined in the following sections. $EDUC\_INF(i,t)$ is the level of public investment in the educational system. CFH The level of the per capita consumption at current price is φ_{EDQUAL}, φ_{wage_prem1}, φ_{wage_prem2}, φ_{MDG_4}, φ_{EDUC_INF} and φ_{C^H} are the elasticities of *SHedint* according to respectively: the quality of the education, education prima, the level of the fourth MDG, the educational investment and the per capita consumption.

The residual behavior

Every year and within every cycle, three cases arise: the students succeed in the final examinations *(grd)* or they fail and repeat *(rep)* or leave the educational system *(dropout)*. Supposing that the proportion of repeaters and expelled remains constant in time, the part of each of these two categories is then given by equations (101) and (102). For the student that obtained a diploma, two scenarios arise: they progress to the following cycle *(grdcont)* or they leave the educational system and join the labor market *(grdexit)*. On the other hand, the share of students that continue in next cycle after graduating , $SHed(\text{grdcont},c,t)$, is given by equation (98). We can then define the share of students that leave the educational system after finishing their studies in the current cycle by equation (103).

$$SHed(\text{dropout}, c, t) = (1 - SHed(grd, c, t))\frac{SHed(\text{rep}, c, t_0)}{SHed(\text{rep}, c, t_0) + SHed(\text{dropout}, c, t_0)} \tag{101}$$

$$SHed(rep, c, t) = (1 - SHed(\text{grd}, c, t))\frac{SHed(\text{rep}, c, t_0)}{SHed(\text{rep}, c, t_0) + SHed(\text{dropout}, c, t_0)} \tag{102}$$

$$SHed(\text{grdexit}, c, t) = (1 - SHed(\text{grdcont}, c, t)) \tag{103}$$

The student number within each cycle

The number of student within a cycle c $N(c,t)$ is defined as the sum of the new students $N^{New}(c,t)$ and the number old students $N^{Old}(c,t)$ (equation 104). The number of new students within a cycle is given by equation (105).

$$N(c,t) = N^{New}(c,t) + N^{Old}(c,t) \tag{104}$$

$$N^{New}(c,t) = \sum_{c'} \delta_{c,c'} SHed(gdrcont, c', t-1) N(c', t-1) + SHed(g1entry, c', t-1) Pop(6,t) + Extra(c,t) \tag{105}$$

$\delta_{c,c'}$ is a Dirac function which is equal to 1 if the cycles c and c' succeed one another and 0 otherwise. $Pop(6,t)$ is the size of the population that has the age of entrance to school. $Extra(c,t)$ is the size of the population that between to cycle c from the outside of the education system (literacy course for example). The number of old students in cycle c at period t is given by equation (106).

$$N^{Old}(c,t) = SHed(concyc,c,t-1)N(c,t-1) + SHed(rep,c,t-1)N(c,t-1) \qquad (106)$$

$SHed(concyc,c,t-1)$ is the share of students that have succeeded but have not obtained their diplomas yet. This share is the difference between the share of students that have succeeded $SHed(grd,c,t)$ and the share of students that have obtained their diplomas $SHed(grdcyc,c,t)$. Equation (107) gives a mathematical expression of this definition. While, the share of students that have obtained their diplomas is given by equation (108).

$$SHed(concyc,c,t) = SHed(grd,c,t) - SHed(grdcyc,c,t) \qquad (107)$$

$$SHed(grdcyc,c,t) = \frac{SHed(grd,c,t)}{Nyear(c)} \qquad (108)$$

$Nyear(c)$ is the number of years of studies within cycle *c*.

The level of MDG 2

The value of the second MDG is defined as the percentage of individuals that have the age to be in the first cycle of the primary education and who are actually there. The number of years of study of the first cycle of the primary education can vary from one country to another and it is denoted *T*, the value of MDG 2 can be expressed by equation (109).

$$MDGVAl(mdg2,t) = SHed(g1entry,edup1,t-T)\prod_{t'=0}^{T} SHed(\text{g1entry},\text{edup1},t-t') \qquad (109)$$

Fight against infant mortality

This objective tries to reduce by two-thirds the mortality rate of children less than five years between 1990 and 2015. In this model we follow Bourguignon et al. (2004) hypothesis stipulating that fight against infant mortality could essentially be won through public health spending. The value of the MDG is given by equation (100).

$$MDGVAL(mdg3,t) = exmdg_{mdg3} + \frac{\alpha_{LOG}(mdg3)}{1+\gamma mdg(mdg3)\exp(\beta_{mdg}(mdg3)\left(MDG\text{int}(mdg3,t) - MGDGval(mdg3,t_0)\right))} \quad (110)$$

$exmdg_{mdg3}$ is the best MDG3 value observed in the world. $\alpha_{LOG}(mdg), \gamma mdg(mdg)$ and $\beta_{mdg}(mdg)$ are the parameters of the Logit function and $MDG\text{int}(mdg,t)$ is an intermediate variable that depends on the levels of public spending affecting this MDG such as the level of public investment, the other MDGs and the level of per capita consumption. The value of $MDG\text{int}(mdg,t)$ is given by equation (111).

$$MDG\text{int}(mdg3,t) = \alpha_m(mdg3)\left[\frac{\frac{GHlth(t)}{Pop(t)}}{\frac{GHlth(t-1)}{Pop(t-1)}}\right]^{\rho_{\text{hlth}(mdg3)}}\left[\frac{\frac{GINF(t)}{Pop(t)}}{\frac{GINF(t-1)}{Pop(t-1)}}\right]^{\rho_{\text{INF}(mdg3)}} \left[\frac{MDGVAL(mdg7a,t)}{MDGVAL(mdg7a,t-1)}\right]^{\rho_{\text{mdg}7a(mdg3)}}\left[\frac{MDGVAL(mdg7b,t)}{MDGVAL(mdg7b,t-1)}\right]^{\rho_{\text{mdg}7b(mdg3)}}\left[BUDC(t)\right]^{\rho_{BUDC(mdg3)}} \quad (111)$$

$GHlth(t)$ being the level of public spending in health at period *t*, $GINF(t)$ the level of public investment at period *t*, *Pop(t)* the size of the population at period *t*, $BUDC(t)$ the households budget assigned to consumption and *MDGVAL(mdg7a,t)* and *MDGVAL(mdg7b,t)* the levels of MDG 7a and MDG 7b as defined by equation (114).

Reduction of maternal mortality

This objective aims at reducing by three quarters the rate of maternal mortality between 1990 and 2015. As the previous objective, maternal mortality is supposed to be strictly connected to the public spending on health. The value of the MDG is given by equation (112).

$$MDGVAL(mdg4,t) = exmdg_{mdg4} + \frac{\alpha_{LOG}(mdg4)}{1+\gamma mdg(mdg4)\exp(\beta_{mdg}(mdg4)\left(MDGint(mdg4,t) - MGDGval(mdg4,t_0)\right)} \tag{112}$$

$exmdg_{mdg4}$ is the best MDG4 value observed in the world. $\alpha_{LOG}(mdg4), \gamma mdg(mdg4)$ and $\beta_{mdg}(mdg4)$ are the parameters of the Logit function and $MDG\,\text{int}(mdg4,t)$ is an intermediate variable that depends on levels of public spending affecting this MDG such as the level of public investment, the other MDGs and the level of per capita consumption. The value of $MDG\,\text{int}(mdg,t)$ is given by equation (113).

$$MDGint(mdg4,t) = \alpha_m(mdg4)\left[\frac{\frac{GHlth(t)}{Pop(t)}}{\frac{GHlth(t-1)}{Pop(t-1)}}\right]^{\rho_{\text{hlth}(mdg4)}}\left[\frac{\frac{GINF(t)}{Pop(t)}}{\frac{GINF(t-1)}{Pop(t-1)}}\right]^{\rho_{\text{INF}(mdg4)}}$$

$$\left[\frac{MDGVAL(mdg7a,t)}{MDGVAL(mdg7a,t-1)}\right]^{\rho_{\text{mdg}7a(mdg4)}}\left[\frac{MDGVAL(mdg7b,t)}{MDGVAL(mdg7b,t-1)}\right]^{\rho_{\text{mdg}7b(mdg4)}}\left[BUDC(t)\right]^{\rho_{BUDC(mdg4)}} \tag{113}$$

Sustainable development

This objective tries to integrate the principles of sustainable development into national policies and to reverse the current tendencies that lead to the depreciation of environmental resources. This objective consists of a series of targets. In this model we limit ourselves to two, namely: access to drinking water and access to sanitary system. As with the preceding objectives, we shall make the hypothesis that these objectives are linked to the levels of public investments in sanitary infrastructures. The values of MDG 7a and 7b are given by equation (114) and (115).

$$MDGVAL(mdg7,t) = exmdg_{mdg7} + \frac{\alpha_{LOG}(mdg4)}{1+\gamma mdg(mdg7)\exp(\beta_{mdg}(mdg7)\left(MDG\text{int}(mdg7,t)-MGDGval(mdg7,t_0)\right)} \quad (114)$$

$$MDGint(mdg4,t) = \alpha_m(mdg4)\left[\frac{\frac{GSAN(t)}{Pop(t)}}{\frac{GSAN(t-1)}{Pop(t-1)}}\right]^{\rho_{SAN(mdg4)}}\left[\frac{\frac{GINF(t)}{Pop(t)}}{\frac{GINF(t-1)}{Pop(t-1)}}\right]^{\rho_{INF(mdg4)}}\left[BUDC(t)\right]^{\rho_{BUDC(mdg4)}} \quad (115)$$

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3. Conclusion

This paper has presented in a detailed manner the properties and the functions of a single country CGE model. This model contributes to the recent debate on the Millennium Objectives and the need for African countries to reorient their economic policies with the view to achieving the MDGs. The most important innovation of this model is that it attempts to go beyond using welfare expenditure considerations as the unique tool for poverty reduction. This model tries to link poverty reduction and the MDGs to economic growth and economic diversification. This will allow African countries to go out of the trap of commodities production and to become more competitive in the global economy.

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