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Multidisciplinary Regional Advisory Group

THE USE OF INPUT-OUTPUT MODELS
FOR ECONOMIC ANALYSIS AND
DEVELOPMENT PLANNING

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I. INTRODUCTION:

During the 1980s, the focus on short-term macroeconomic management under the auspices of the IMF/World Bank sponsored structural adjustment and stabilization programmes led to the complete neglect of long-term macro-economic planning in the African countries. This deliberate abandonment of systematic macro-economic planning implied that there was no coherent and consistent policy for long-term socio-economic transformation and sustained development of the African economies.

Consequently, if the much desirable goal of sustained economic growth with transformation is to be realized in the 1990s, it is absolutely necessary to return to a situation where both short-term macro-economic management of the brand initiated by the World Bank and the International Monetary Fund in many African countries; and long-term development planning as was symbolized by elaborate Five Year Development Plans during the early 1960s, are undertaken, properly coordinated and harmonized.

There is no doubt that a dynamic planning process (as opposed to the traditional plan formulation or an over-centralized planning system) is essential for bringing together all the socio-economic actors as well as the institutions responsible for steering the national economy (Ministries of Planning and Finance and sectoral Ministries, the private sector and NGOs) and for providing the policy-makers with relevant and operational development options. To this extent, the planning process can serve as an instrument for promoting a national dialogue on long-term socio-economic development strategies and societal choices for the future.
Such choices can only be made after a thorough macro-economic analysis and the implications of alternative policy options on the economy and society. In this regard, the importance of macro-economic models, as quantitative techniques of planning, in the identification of appropriate ways of achieving agreed national objectives and defining realistic alternative choices based on present constraints and future imperatives, cannot be overemphasized.

In view of the importance of development planning as an effective instrument for helping African policy-makers marshall the strategic factors which influence the socio-economic evolution and transformation of their countries as well as manage coherently their development programmes and projects, the purpose of this paper is to examine the usefulness of input-output models in economic analysis and planning. Specifically, the paper attempts a definition of an input-output table, examines a typical input-output table, presents a hypothetical example, examines the underlying assumptions of input-output analysis, presents a mathematical input-output model, analyses the sector limitations and finally the expansions of the basic input-output model.

It is hoped, therefore, that the paper will serve as a catalyst in providing an opportunity for Ethiopian planners and practitioners to exchange their experiences as well as discuss the methodological and operations issues involved in the national development planning of the country. In addition, it is also hoped that the exercise will lead to the formulation of a common vision on what should be done to ensure rapid socio-economic transformation and sustained development in Ethiopia.

II. DEFINITION OF AN INPUT-OUTPUT TABLE

An Input-output table can be regarded as a collection of data describing the particular structural characteristics of any
economic system. It can also be regarded as an analytical technique for explaining and influencing the behavior of a system at a certain point in time or over the course of time. The basic notion of Input-Output analysis rests on the belief that the economy of any country can be divided into a distinct number of sectors called industries (or sometimes activities), each consisting of one or more firms producing similar but not necessarily homogenous products.

Each industry requires certain inputs from other sectors in order to produce its own output. Similarly, each industry sells some of its gross output to other industries so that they too can satisfy their intermediate material needs. The Input-Output table provides a convenient framework for measuring and determined these interindustry blows of current inputs and outputs among the various sectors of the economy.

III. PRESENTATION OF A TYPICAL INPUT-OUTPUT TABLE

In Table 1, we have a representation of the structural make up of a typical Input-Output table. Most of the information contained in the table is located within the three main quadrants (a) the "Interindustry Transaction" quadrant (II); (b) the "Value-Added quadrant (III); and (c) the "Final Use" quadrant (I).
### Table 1: The Make-Up of a Typical Input-Output Table

<table>
<thead>
<tr>
<th>Sector 1</th>
<th>Sector 2</th>
<th>Sector 3</th>
<th>Sector 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outputs</td>
<td>Outputs</td>
<td>Outputs</td>
<td>Outputs</td>
</tr>
<tr>
<td>Inputs</td>
<td>Inputs</td>
<td>Inputs</td>
<td>Inputs</td>
</tr>
<tr>
<td>Value Added</td>
<td>Value Added</td>
<td>Value Added</td>
<td>Value Added</td>
</tr>
<tr>
<td>Direct Factor Purchase</td>
<td>Direct Factor Purchase</td>
<td>Direct Factor Purchase</td>
<td>Direct Factor Purchase</td>
</tr>
<tr>
<td>Final Use</td>
<td>Final Use</td>
<td>Final Use</td>
<td>Final Use</td>
</tr>
<tr>
<td>Value Added</td>
<td>Value Added</td>
<td>Value Added</td>
<td>Value Added</td>
</tr>
<tr>
<td>Final Use Quadrant (I)</td>
<td>Final Use Quadrant (I)</td>
<td>Final Use Quadrant (I)</td>
<td>Final Use Quadrant (I)</td>
</tr>
<tr>
<td>Intermediate Use</td>
<td>Intermediate Use</td>
<td>Intermediate Use</td>
<td>Intermediate Use</td>
</tr>
<tr>
<td>Total Intermediate Use</td>
<td>Total Intermediate Use</td>
<td>Total Intermediate Use</td>
<td>Total Intermediate Use</td>
</tr>
</tbody>
</table>

Total Output of Sector i = Total Inputs of Sector i - Total Output of Sector i - Total Input of Sector i, ∀ i = 1, 2, …, n.
The 'Direct Factor Purchase' quadrant (IV) is not as important for planning purposes as the other three although it is necessary for accounting purposes, especially for measuring the GDP.

The producing sectors of the economy are listed as rows 1, 2, \ldots N, in the transaction quadrant (II) while these same industries are also listed by columns as using sectors. The transaction quadrant is thus always a square matrix with the same number of rows as columns, one for each sector of the economy. In the case of the Final Use quadrant (I) we see that the outputs of each sector are also normally demanded for ultimate use in the form in which they are produced, e.g. as final demand products which are consumed for their own sake and not used for further production. They may be purchased as consumption goods by individual consumers or by the government. They may also be purchased as investments goods by the state or by private investors if we are dealing with a mixed market economy or they may be sold to foreigners in the form of exports.

Reading down columns 1 to N in the 'Value Added' Quadrant (III), we obtain information about the amount of primary inputs (i.e. land, labor, capital and natural resources) used by each sector in the production of its outputs. The sum of the value of these primary inputs used in the whole economy yields the total value added by industries. For any particular sector, the elements in each column of this quadrant when added to the elements of the corresponding column of the transactions quadrant immediately above yield a value for total inputs purchased or used up by this industry during the accounting period, i.e. the total costs of operation.

Finally, the "Direct Factor Purchase" quadrant (IV) shows those primary inputs which are employed by final users. The main component of this quadrant would be the purchase of labor services by the government. For example, in most African economies
government employment in the various ministries and branches of civil service is ordinarily quite substantial. Total government expenditures for labor services would, therefore, be entered in this quadrant in the box where the labor row intersects the government consumption column. Households also purchase labor in the form of domestic help and the value of such activities would be entered accordingly at the intersection of the labor row and the private consumption column. The sum of all entries in quadrants III and IV yields an alternative figure for gross domestic product - GDP.

IV. A HYPOTHETICAL EXAMPLE

Let us now turn to an input-output table using hypothetical data and review the above procedures with the aid of numerical figures. Table 2 provides a numerical illustration of the basic input-output accounting system. For simplicity, we have divided our table into only 5 sectors (agriculture, sector 1, extractive industry - sector 2, manufacturing industry - sector 3, power - sector 4 and transportation - sector 5). Note that each sector appears in the accounting system twice, as a producer of outputs (rows 1-5) and as a user of inputs (columns 1 - 5).

The elements in each row show a particular sector despaired of its total output during a given accounting period. For example, of the total available output of agricultural products (125 units) 15 units are used by agriculture itself, 20 by manufacturing and 10 by transportation. The total intermediate use of agricultural products i.e. use for further production is 45 units. To this figure, must be added the quantity of agricultural goods demanded by final users which in Table 2 consist of consumption of 35 units by household, 15 units by government 30 units exported to foreign countries, and 5 units for non-estimated expenditures.
Using sector (input)
Producing Section (Output)

<table>
<thead>
<tr>
<th>Department</th>
<th>Agriculture</th>
<th>Extractive Industry</th>
<th>Manufacturing</th>
<th>Power</th>
<th>Transportation</th>
<th>Total Purchases</th>
<th>Total Imports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value Added</td>
<td>105</td>
<td>105</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>45</td>
<td>15</td>
</tr>
<tr>
<td>Government</td>
<td>105</td>
<td>105</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>45</td>
<td>15</td>
</tr>
<tr>
<td>Households</td>
<td>105</td>
<td>105</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>45</td>
<td>15</td>
</tr>
<tr>
<td>Capital (C)</td>
<td>105</td>
<td>105</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>45</td>
<td>15</td>
</tr>
<tr>
<td>Natural Resources (N)</td>
<td>105</td>
<td>105</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>45</td>
<td>15</td>
</tr>
<tr>
<td>Value Added</td>
<td>105</td>
<td>105</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>45</td>
<td>15</td>
</tr>
<tr>
<td>Government</td>
<td>105</td>
<td>105</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>45</td>
<td>15</td>
</tr>
<tr>
<td>Households</td>
<td>105</td>
<td>105</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>45</td>
<td>15</td>
</tr>
<tr>
<td>Capital (C)</td>
<td>105</td>
<td>105</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>45</td>
<td>15</td>
</tr>
<tr>
<td>Natural Resources (N)</td>
<td>105</td>
<td>105</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>45</td>
<td>15</td>
</tr>
</tbody>
</table>

**Table II**

<table>
<thead>
<tr>
<th>Total Output</th>
<th>Agriculture</th>
<th>Extractive Industry</th>
<th>Manufacturing</th>
<th>Power</th>
<th>Transportation</th>
<th>Total</th>
<th>Final Use (Demand)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value Added</td>
<td>105</td>
<td>105</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>45</td>
<td>15</td>
</tr>
<tr>
<td>Government</td>
<td>105</td>
<td>105</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>45</td>
<td>15</td>
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<tr>
<td>Households</td>
<td>105</td>
<td>105</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>45</td>
<td>15</td>
</tr>
<tr>
<td>Capital (C)</td>
<td>105</td>
<td>105</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>45</td>
<td>15</td>
</tr>
<tr>
<td>Natural Resources (N)</td>
<td>105</td>
<td>105</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>45</td>
<td>15</td>
</tr>
</tbody>
</table>

**Note:**
- The table represents the hypothetical input-output table for various sectors.
- The values in the table are illustrative and do not represent real data.
The sum of total intermediate and total final demand yields a gross output for agricultural production of 125 units. Similarly we see that the extractive sector produces a total of 40 units of which none is sold on an industry basis to other sectors while the government purchases 10 units and 30 units are exported. The disposition of the total outputs of the other three sectors can be read off the table in the same manner.

In terms of national income accounting techniques we can arrive at identical figures for GDP by using either the income or the product approach. Using the income value added approach GDP is determined as the summation of total payments (both intermediate and final) to primary inputs plus business tax payments. In table 2, we observe that these total payments amounted to 158 units. Using the product approach which defines GDP as the difference between total final demand \((80 + 40 + 45 + 15 + 15 = 205)\) and intermediate imports \((60)\) we arrive at the same figure of 158 units for GDP.

The Input-Output table can be used to show the same (row 8) and uses (columns 8 and 9) of government funds or the current account of balance of payments e.g. exports (column 10) minus imports (row 7) would yield a net 'deficit' of 5 units (i.e. 65-70 = -5) from our hypothetical table.

It should be noted that the major theoretical and practical value of Input-Output tables is that they can easily be transformed into a consistent mathematical model and utilized as a forecasting tool to predict the effects of autonomous changes in final demand on total output and employment in all sectors of the economy or as in centralized socialist economies, as a framework within consistent comprehensive economic plans can be drawn up and carried out. In order to transform the accounting table into a workable mathematical model, however, certain necessary assumptions about the production process must be made.
V. UNDERLYING ASSUMPTIONS OF INPUT-OUTPUT ANALYSIS

The underlying assumption of the input-output analysis which makes the system operationally effective, is that a single process of production function exists in every industry. This assumption can be broken down into a number of sub-operationeal constituent parts as follows: (i) there are constant returns to scale; and (ii) there is no substitution among inputs in the production of any good or service. In other words, since there is only one process or method of production in each industry, the level of output of a product determines uniquely the level of each input required.

Technically, we may say that the production process is assumed to be characterized by the existence of "constant technical coefficients of production" i.e. each additional unit of new output is produced by an unchanging proportional combination of material inputs from other sectors. For example in Table 2, we can observe that in order to produce one hundred units of agricultural output for its intermediate input needs. Dividing 20 by 100 we find that our proportional assumption indicates that for every unit of manufacturing output, 0.20 units of agricultural products will always be required as inputs so long as manufacturing sector remains unchanged by the prevailing technology.

(iii) Each sector produces a well defined commodity or group of commodities which are not produced by any other sector.

(iv) At a point in time, each sector uses only one method of production, which implies that the set of inputs per unit of output is unique.

(v) It is only the level of output of a sector that determines the magnitudes of inputs from other sectors.
(vi) As a result of assumptions (iii) and (v), the total effect of undertaking several types of production, is the sum of separate effects. This implies aggregation of productive activity and rules out economies and diseconomies of scale.

(vii) Any Input-Output system with only one coefficient for each combination of an input does not allow for any substitution of inputs and there is only one set of equilibrium prices associated with it. This is true even when the components of final use change.

VI. A MATHEMATICAL INPUT-OUTPUT MODEL

The analytical and mathematical content of the Input-Output model rests on a dual foundation. The first consists of set of accounting equations; one for each producing sector of the economy. The first of these equations states that the total output of sector 1 is equal to the sum of the separate amounts sold by sector 1 to other industries plus the amount produced to satisfy final demands. The second equation says the same thing for sector 2 - and so on for all industries.

In terms of our Input-Output table, these equations state that for any sector, total output is equal to the sum of all the entries in that sector’s row in the table. Thus, an implicit assumption of Input-Output analysis common to all general equilibrium models is that in all sectors the entire product is consumed either by other industries as intermediate inputs or by final users. In short, supply always equal demand.

We can state this first equation in concise symbolic terms. To this extent, let $X_i$ measure the annual rate of total output (in the appropriate value units) of industry $i$; $X_{ij}$ represent the amount of the product of industry $i$ absorbed annually as an intermediate input by industry $j$; and finally let $Y_i$ equal the amount of the same product $i$ produced to satisfy final demand.
The overall Input-Output accounting balance for the entire economy comprising $n$ separate industries or sectors can now be described in terms of $n$-linear functions:

1) $\sum_{j=1}^{n} X_{ij} + Y_i = X_i$

where $i = 1, 2, \ldots, n$.

Each equation is representative of the fact that in all sectors the entire product produced ($X_i$) is consumed either by the other industries $\sum_{j=1}^{n} X_{ij}$ or by final demand $Y_i$. For example in Table 2, we would have such accounting equations. In the case of industry 3 (i.e. manufacturing) the accounting equation will be of the following order:

$\sum_{j=1}^{5} X_{3j} + Y_3 = X_3$

or substituting the appropriate numerical figures, we have $10 + 0 + 25 + 15 + 5 + 45 = 100$

The second and more important foundation of the Input-Output model consists of another set of $n$-equations; one for each industry, describing the Input-Output structure of that industry in terms of a derived set of $a_{ij}$ technical coefficients of production. Thus, the commodity flows $X_{ij}$, included in the above balance equations are subject to the following set of structural relationships:

2) $X_{ij} = a_{ij} X_{j1}$

$i = 1, 2, \ldots, n$

$j = 1, 2, \ldots, n$

Substituting for $X_{ij}$ from equation (2) into equation (1) and transposing the terms, we obtain the following expression:
(3) \[ X_i - \sum_{j=1}^{n} a_{ij}X_j = Y_i \]

Where \( i : 1, 2, \ldots, n \).

In terms of our hypothetical economy, system (3) would consist of 5 linear equations that would be written symbolically and numerically as follows:

\[
\begin{align*}
X_1 &= a_{11}X_1 - a_{12}X_2 - a_{13}X_3 - a_{14}X_4 - a_{15}X_5 = Y_1 \\
X_2 &= a_{21}X_1 - a_{22}X_2 - a_{23}X_3 - a_{24}X_4 - a_{25}X_5 = Y_2 \\
X_3 &= a_{31}X_1 - a_{32}X_2 - a_{33}X_3 - a_{34}X_4 - a_{35}X_5 = Y_3 \\
X_4 &= a_{41}X_1 - a_{42}X_2 - a_{43}X_3 - a_{44}X_4 - a_{45}X_5 = Y_4 \\
X_5 &= a_{51}X_1 - a_{52}X_2 - a_{53}X_3 - a_{54}X_4 - a_{55}X_5 = Y_5
\end{align*}
\]

Numerical representation is as follows:

\[
\begin{align*}
125 &= .12 (125) - 0 (40) - .20 (100) 0 (75) - .20 (50) = 80 \\
40 &= 0 (125) - 0 (40) - 0 (100) 0 (75) 0 (50) = 40 \\
100 &= .08 (125) 0 (40) - .25 (100) - .20 (75) - .10 (50) = 45 \\
75 &= .04 (125) - .375 (40) - 15 (100) - 0 (75) - .30 (50) = 25 \\
50 &= .04 (125) - .25 (40) - .15 (100) - 0 (75) - .10 (50) = 15
\end{align*}
\]

For convenience let us rewrite system (3) in terms of matrix and vector notations as follows:

\[(4) \quad X(A) \quad X = Y, \quad \text{where} \quad X \text{ represent a column vector of total outputs consisting of n elements. In our example n = 5, each of which numerically represents the total output of one of the n industries.}\]

\( (A) \) is a n\times n square matrix of technical coefficients of production described earlier; and \( (Y) \) is a column vector of total
final demands. Thus, if the economy were divided into 30 sectors (i.e. \( n = 30 \)), system (4) would be a convenient way of avoiding the tedious task of writing out a set of 30 simultaneous equations.

VII. INTERDEPENDENCE OF SECTORS AND GROUPS OF SECTORS

In Input-Output analysis, a system of general interdependence in which each sector has a direct connection with every other sector is generally assumed. In practice, it is observed that this inter-dependence of industries is limited. Some industries may draw their inputs from a limited range of other industries and some groups of industries may tend to form blocks with a great deal of buying and selling within groups but relatively between groups.

Chenery and Clark (Inter-Industry Economics) have discovered that by an appropriate adjustment and interchange of sectors, Input-Output tables can be approximated roughly by a triangular form. The pattern they have attempted to fit to the interindustry flows of each country is that of one-way interdependence, as can be described by the system of equations (1) for five sectors of economy as follows:

\[
\begin{align*}
X_1 &= a_{11}X_1 + \varnothing_1 \\
X_2 &= a_{21}X_1 + a_{22}X_2 + \varnothing_2 \\
X_3 &= a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + \varnothing_3 \\
X_4 &= a_{41}X_1 + a_{42}X_2 + a_{43}X_3 + a_{44}X_4 + \varnothing_4 \\
X_5 &= a_{51}X_1 + a_{52}X_2 + a_{53}X_3 + a_{54}X_4 + a_{55}X_5 + \varnothing_5
\end{align*}
\]

where \( \varnothing \)'s are plan targets in physical terms.

If an Input-Output table can be represented in a triangular form as above, with values of the elements above the diagonal being
very insignificant or zeros, the values of total output of sectors affected by a change in a component of the final deliveries can be easily derived. Whenever a sector’s final delivery has changed only this sector and those recorded below it will have altered outputs.

As the outputs of sectors appearing above this sector are known and unaffected, the change in the output whose final delivery has changed can be immediately obtained. And by successive substitution, the changed outputs of the remaining sectors can be estimated without any need for inverting the matrix. Further, since the policy changes introduced in a sector do not affect the sector appearing earlier in the triangular pattern the effects of a policy change in a certain sector need to be traced only in respect of the sectors lying below it.

If the groups of sectors formed completely independent blocks without any interlock deliveries, then the computing work of input-output analysis is again greatly simplified. Let us consider an economy of five sectors, the first three sectors are completely independent of the last two sectors:

\[
\begin{align*}
  X_1 &= a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + \varnothing_1 \\
  X_2 &= a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + \varnothing_2 \\
  X_3 &= a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + \varnothing_3 \\
  X_4 &= a_{44}X_4 + a_{45}X_5 + \varnothing_4 \\
  X_5 &= a_{54}X_4 + a_{55}X_5 + \varnothing_5 
\end{align*}
\]

System of equations (2) above can be divided into two completely independent systems since there are no intersectoral deliveries between the first three sectors and the last two sectors. The two structures are completely independent and can be solved separately. Each of them is easier to solve and analyse. Though in reality completely independent blocks of sectors in an economy seldom exist, an underdeveloped economy can be roughly divided into two blocks, one the traditional agricultural sector and the other the non-traditional or modern industrial sector.
VIII. SECTOR LIMITATIONS

Individual sectors of an input-output system may be subject to various types and degrees of limitations. We shall discuss here two extreme cases. First is the one in which a sector need not expand or contract with the system, that is, the output level of the sector may not have to change with the vector of final use.

The second, is the case in which the output level of a sector cannot be increased beyond a certain level.

In the first case, sectors are divided into two parts, one including those whose outputs are freely imported and exported and the other including the remaining sectors whose outputs cannot be imported. The sectors belonging to the first category are called "international sectors" and the other, "national sectors". A change in the requirements of goods produced by the international sectors may not require a change in the level of operation of international sectors but a change in the requirements of goods produced by the national sector must be accompanied by a corresponding change in the level of operation of these sectors.

Let us suppose that in our example, the first three sectors are "international" and the last two are "national". Since the goods of the international sector can be imported, the production level of those sectors to be produced domestically can be fixed. However, the production level of national sectors for meeting the domestic production of goods of national as well as international sectors has to be realized. If we assume that the levels of output of the international sectors to be produced domestically are fixed at $X_1^*, X_2^*, X_3^*$, the balance equations for the national sectors can be written as follows:

\[
\begin{align*}
X_4 &= a_{41}X_1^* + a_{42}X_2^* + a_{43}X_3^* + a_{44}X_4 + a_{45}X_5 + \phi_4 \\
X_5 &= a_{51}X_1^* + a_{52}X_2^* + a_{53}X_3^* + a_{54}X_4 + a_{55}X_5 + \phi_5
\end{align*}
\]
Solving the above equations for $X_4$ and $X_5$, and the levels of output of the domestic sectors compatible with the production levels of international sectors can be obtained. The advantage of this method is that it allows freedom in fixing the output levels of international sectors and thus facilitates the planning of national sectors. In standard input-output models, if one sector expands, other sectors must also expand correspondingly, and there is no mechanism to reduce the exports of the international sectors so as to increase the expansion of domestic sectors. The above method relaxes the rigidity of common input-output models.

The second case relates to a situation which is often met in practical planning. The capacity of some sectors is limited because of physical or technical conditions in the economy, and cannot produce more than certain maximum levels of output. If the capacity of such sectors is known and given, the total output levels of other sectors and the final deliveries of the capacity limited sectors can be derived, given the final demand for the non limited sectors.

As an illustration, let us suppose that the first three sectors of our five sector economy are limited by given capacity outputs of $X_1$, $X_2$, and $X_3$. Let the amount of output for final use for the remaining two unlimited sectors be $\phi_u$ and $\phi_{5u}$. The total output of the unlimited sectors can be written as follows:

\[
(2) \quad X_{4u} = a_{41}X_1 + a_{42}X_2 + a_{43}X_3 + a_{44}X_4 + a_{45}X_5 + \phi_{4u}
\]

\[
X_{5u} = a_{51}X_1 + a_{52}X_2 + a_{53}X_3 + a_{54}X_4 + a_{55}X_5 + \phi_{5u}
\]

This can be solved for $X_{4u}$ and $X_{5u}$. The levels of final use of the capacity limited sectors are:

\[
(3) \quad \phi_1 = X_1 - a_{11}X_1 - a_{12}X_2 - a_{13}X_3 - a_{14}X_4 - a_{15}X_5
\]

\[
\phi_2 = X_2 - a_{21}X_1 - a_{22}X_2 - a_{23}X_3 - a_{24}X_4 - a_{25}X_5
\]

\[
\phi_3 = X_3 - a_{31}X_1 - a_{32}X_2 - a_{33}X_3 - a_{34}X_4 - a_{35}X_5
\]
It may turn out that one or more of $\phi_1$, $\phi_2$ and $\phi_3$ are negative or less than the minimum amount required for final use as a target. Then, if these deficits cannot be met from imports, the final deliveries of unlimited national sectors may have to be adjusted to allow for the residue of minimum final demand required from the capacity limited sectors.

A straightforward application of input-output analysis ignores the capacity limitations of individual sectors. An application of the above modified procedure will make the estimation of the volumes of sectoral outputs as well as the setting of plan targets more realistic.

IX. SOME EXPANSIONS OF THE BASIC I-O MODEL

Before concluding the discussion of the Static I-O Model mention should be made of a few of the other important areas of economic analysis where the model can be extremely useful. Some of these areas are the determination of employment levels and the impact of a planned change in total output on the balance of payments.

(a) Determination of Employment Levels

In using the I-O basic model to estimate the impact of any change in final demand or total output on the level of total industrial employment in the economy, it is generally assumed that such employment is distributed in certain proportions throughout all industries. In the household row of primary inputs in Table 2, we have a set of five figures representing the value of labor input used by each of the five sectors. If these values are all written in terms of some average wage rate in the economy, then the relative magnitude of the various figures would correspond to the relative number of workers employed in that sector.
Assuming that the average annual wage rate in our hypothetical economy was equal to 0.001 units of value and the household figures presented in Table 2 represented the total value of manufacturing years employed by each sector, then we could say that agriculture is employing 40 000 Man-years of labor (i.e. \(40 \times 1 = 40\ 000\)); extractive industries 5 000 Man-year, and manufacturing, power and transportation 6 000, 5000 and 2 000 man-year, respectively. Total employment in the five sectors is thus at a level of 58 000 Man-years. Since our I-O table is based on an accounting period of one year, we may refer to a total employment of 58 000 workers.

In a manner analogous to the derivations of our technical coefficient, \(l_i(i = 1, 2, \ldots, n)\) each element of which depicts the number of workers (or man years of employment) required to produce a unit of industry \(i\)'s output. The labor coefficient is thus calculated as follows for each industry:

\[
L_i = \frac{l_i}{X_i} \quad i = 1, 2, \ldots, n.
\]

Where \(l_i\) is level of employment in industry \(i\) and \(X_i\) is again total output of industry \(i\). For example, the labor coefficient for our five sectors of economy would be as follows:

\[
L_1 = \frac{40}{125} = .32
\]

\[
L_2 = \frac{5}{40} = .125
\]

\[
L_3 = \frac{6}{100} = .060
\]

\[
L_4 = \frac{5}{75} = .066
\]

\[
L_5 = \frac{2}{50} = .040
\]
The level of employment in each industry is uniquely related to the amount of total output produced in that industry. Thus, to find the amount of labor employed in industry \( i \), we merely multiply the corresponding labor coefficient \( l_i \) by the total output \( X_i \) of that sector. By summing the products of labor coefficient and total outputs of all industries throughout the economy, we can derive the following expressions for full employment:

\[
L_T = \sum_{i=1}^{n} l_i \Delta X_i
\]

where \( L_T \) represents total industrial employment in the economy. Similarly, a change in employment as a result of a change in total output can be expressed as:

\[
\Delta L_T = \sum_{i=1}^{n} l_i \Delta X_i.
\]

Finally, since we know that \( \Delta X_i = \sum_{j=1}^{n} r_{ij} \Delta Y_j \), the change in employment as a consequence of any given anticipated planned change in final year demand, can be calculated by substituting this relationship into equation (2). Thus, we have the following expressions:

\[
\Delta L_T = \sum_{i=1}^{n} l_i \left( \sum_{j=1}^{n} r_{ij} \Delta Y_j \right)
\]

For example, the change in total demand that would result directly and indirectly from our hypothetical increase in the full demand for the products of the manufacturing sector (i.e. \( \Delta Y_3 = +10 \)) would be

\[
\Delta L_T = 1_r 1_{13} \Delta Y_3 + 1_{22} r_{23} \Delta Y_3 + 1_{32} r_{33} \Delta Y_3 + 1_{43} r_{43} \Delta Y_3 + 1_{53} r_{53} \Delta Y_3
\]

\[
= .32 (.40 \times 10) + .125 (0 \times 10) + .06 (1.5 \times 10) + .062 (.32 \times 10)
\]

\[
+ .04 (.27 \times 10) = 1.28 + 0 + .90 + .211 + .108 = 2.499
\]
Thus, there will be increased employment opportunities for 2,499 workers as a result of 10 units or 22 per cent (10/45 = 22) rise in final demand for manufactured products. It is interesting to note that the greatest individual sector (900 workers) as we might expect is in the agricultural sector where 1,280 new jobs will be created. It is in unexpected circumstances like these that the real value of input-output analysis becomes strikingly evident.

b) Balance of Payment Analysis

The input-output model can be used also in the area of international trade to examine the approximate impact of any predicted or planned change in final demand or total output on the balance of payments (current account) position of the given economy. Since foreign trade is such an integral aspect of the structure of most African economies, any complete and comprehensive development plan must include some estimate of future balance of payments situations.

We can use our row vector of intermediate imports in Table 2 and our column vector to derive a row vector of import coefficient \( M_i = \frac{M_i}{x_i} \) for each sector of the economy. Again the procedure is exactly the same namely:

1. \( M_i = \frac{M_i}{x_i} \) for \( i = 1, 2, \ldots, n \).

where \( M_i \) is equal to the value of intermediate imports of sector \( i \), thus, the intermediate import coefficient for our five sector

\[
\begin{align*}
M_1 &= \frac{15}{125} = .12 \\
M_2 &= \frac{0}{40} = 0 \\
M_3 &= \frac{10}{100} = .10 \\
M_4 &= \frac{30}{75} = .40
\end{align*}
\]
Any change in total output or final demand will lead to an induced change in intermediate imports in accordance with the following two expressions:

\[ \Delta MT = \sum_{i=1}^{n} M_i \Delta x_i \text{ or } (2b) \ \Delta MT = \sum_{i=1}^{n} \sum_{j=1}^{n} M_{ij} \Delta y_j \]

\(\Delta MT\) represents the change in the total value of intermediate imports. For example, as a result of our 10 units increase in final demand for manufactured goods, induced intermediate imports will rise by:

\[ \Delta MT = .12(4) + 0(0) + .10 (15) + .40 (3.2) + .10 (2.7) \]

\[ = .48 + 0 + 1.5 + 1.28 + .27 = 3.53 \text{ units of value.} \]

Unless exports are expanded, the increase in final demand could lead to balance of payment problems. It is from important questions like these that arise out of certain seemingly unrelated circumstances, that planners must direct their attention if the development plan is in any sense to be comprehensive.

So far we have seen that a model depicting the interdependence among two or more main sectors of the economy can assist greatly in the formulation of a development plan that takes account of broad interrelationship. However, when it comes to the formulation of detailed and comprehensive development plans where internally consistent production targets and import requirements are designed on an industry by industry basis; each branch of the broad sectoral model must be broken down even further into its component subsectoral units and analyzed within some variant of the static input-output framework which we have examined earlier.

Although interindustry models are primarily applicable in economies that have achieved a certain degree of industrial development where a considerable volume of statistically recorded
intermediate material transactions takes place, the construction of a simple input-output table can nevertheless, make a significant contribution in countries where industrial activity is still in its infancy. Thus, although practical applications of the type described earlier may be precluded by a lack of data in most African countries, the preparation of a rough input-output table is quite often a useful way to organize the data that do not exist and to locate statistical inconsistencies and deficiencies where further investigation would be of great value.

Furthermore, the existence of many empty boxes; that is zero interindustry transactions, revealed by the rough input-output table provides an excellent means of extracting the total potential contribution that the planned introduction of any new industry can be expected to make on income, output and employment in the overall economy. A related use of input-output table is to serve as an intermediate step in the construction of a detailed system of national income accounts.

X. PRACTICAL USES OF I-O MODELS IN DEVELOPMENT PROGRAMMING

Our analysis of the I-O methodology earlier touched upon some areas of planning in the former command economies where the model was of considerable practical and analytic value. In the field of development programming where the number of producing sectors is relatively small (say 15 to at most 40 major industries in most developing economies) the practical applicability of the I-O techniques assumes a position of paramount importance. With only say, twenty five industries to deal with, the mathematical models are rendered relatively simple since the computation of an inverse matrix for such a table readily attainable with even the most simple variety of modern electronic computer.
Furthermore, many crucial analytic areas of development planning, such as the computation of internally consistent labor, capital and import requirements on the basis of alternative output or investment targets, can be handled with relatively unsophisticated mathematical methods. Let us therefore, examine some of the more important practical uses of the simple dynamic output-input model in the field of development planning.

a) Import Requirements and Substitution Possibilities

While the primary application of I-O in developed centralized economies was to provide a method for arriving at detailed production levels that were consistent with each other and with final demand targets as expressed, for example, by material balance of the former Soviet Plans; the practical utility of input-output in developing economies; oriented as they are towards foreign trade, is derived more from its usefulness in estimating import requirements and evaluating possibilities of import substitution. Consequently, when drawing up an I-O table, for a developing economy, planners often use a codified form of the Static I-O model described earlier.

Rather than a single row for imports and a single column for exports, several rows and columns are often utilized to distinguish among various categories of exports and imports. For example, the import sector could usefully be broken down into separate rows representing: imports of raw materials and intermediate products

- imports of finished products
- imports of capital goods, and
- transfers such as loans and grant for specific projects.

Furthermore for countries of Central Africa where an economic and customs union like the UDEAC of considerable importance has existed since 1964, there may be separate rows and columns for import and export transactions, between say Cameroon and Congo, and
Gabon and Central African Republic. Such a distinction between trade with the rest of the world and other UDEAC member states, would in the Central African situation, greatly help in evaluating the desirability of establishing specific import-substituting industrial projects in any one country and the impact that these projects might have on the economic well being of the entire subregion.

b) Choice Among Export Promotion Alternatives

One often finds in the literature on economic development the argument that in order to overcome their dependence on the fluctuations of primary product export prices, less developed Africa economies might best embark on a vigorous programme of export diversification and export promotion rather than follow a single policy of import substitution. In such circumstances, the input-output methodology can be used to evaluate the impact of alternative export promotion projects on income and employment in the national economy as well as on the levels of anticipated foreign exchange.

It is possible that the foreign trade sector of the input-output table can be disaggregated into individual components reflecting the exact country or currency of origin and destination of exports, in order to obtain a separate 'foreign trade matrix' within which alternative possibilities of export diversification might be ascertained. Given the expected values of any new export activities, one could then analyse the impact of these new industries on output, employment and imports of all other sectors of the economy.

c) Efficient Investment Allocation

One of the main problems of applying the methodology of the "static" input-output model to the formulation of policy and
development planning is that this model fails to take account of the very important phenomenon of economic development; namely, that investment cannot realistically be considered as autonomous final demand like consumption, government spending, and exports. Rather we must recognize that the magnitude of capital accumulation in any one year is itself highly dependent on the level of consumption; government expenditures and especially in Africa, on the value of total exports.

The static input-output model analyzed earlier was based on the assumption that only current flows of inputs and outputs are important aspects of a development plan. Specifically the model assumed that investment can be included as one of the components of an exogenously determined level of final demand for the goods of a particular industry and that its magnitude in any given year is not associated with the level of economic activity in that industry.

By so doing, the static input-output model divorces investment decisions from output objectives and from capacity considerations. Since it is intuitively clear that the level of investment in any sector of a developing economy depends greatly on the degree to which existing capacity is being utilized and that limited capital goods must be used in an efficient way, it becomes necessary to build investment requirements into the basic framework.

By means of a simple refinement of the static model, development planners can "dynamize" the input-output model by including subsectoral capital coefficients which relate investment requirements to continuous changing present and future output targets. The resultant model is known as the "Dynamic Input-Output model."

It should be noted that the introduction of a matrix of subsectoral capital coefficients into the static input-output model recognizes the fact that in addition to the current intermediate
material flow requirements of any economic system (reflected by the matrix of technical coefficients of production); there exists capital or stock requirements (described by a matrix of capital-coefficients) that must be satisfied if necessary buildings, machinery and inventories are to be available to satisfy output objectives. Dynamic input-output analysis thus attempts to take account of capital requirements and economic growth overtime.

XI. INPUT-OUTPUT MODELS AND THE PROBLEM OF STRUCTURAL CHANGE

There is one main aspect of the input-output model which is inconsistent with an important notion of the theory of economic development. This the apparent dichotomy between the input-output assumption of a single-process production function with constant technical and capital coefficients in each sector of the economy and the notion of structural change as an important phenomenon of economic development. Two crucial planning questions emerge from this dichotomy.

First how are we to reconcile output and capital requirement projections based on existing technological relationships with the desire to introduce new methods of production and improved technologies? Second, how are we to consider and treat the establishment of entirely new industries e.g. of the import-substituting variety with the tools of inter-industry economics?

The second problem presents no great conceptual and practical difficulties because it can be handled relatively easily within the existing input-output framework. As new industries emerge and begin engaging in intermediate material and capital transactions with other sectors of the economy, we merely fill in the so called 'empty boxes' of our input-output table by adding another row of intermediate and final outputs and another column of material and factor inputs to represent the activities of this new industry.
We can then compute the relevant technical and capital coefficients of production either on the basis of preliminary engineering information about the structure of production in this new sector or on the basis of statistical experience of similar industries already established in countries of comparable economic development. In addition, whenever the introduction of a new industry results in a change of interindustry transactions, the existing material and capital coefficients will have to be modified accordingly. But this leads us into the more difficult first question of structural change within established industries.

The expectation of significant structural changes which are an inherent manifestation of the process of economic development, introduces a serious complication for the direct application of simple static and dynamic input-output techniques. Two related assumptions of input-output analysis give rise to this problem. These assumptions are: a) the seemingly unrealistic assumption that coefficients of production do not change overtime and b) the highly restrictive assumption that each industry has at its disposal only one method or process reflected (by a single set of technical coefficients) of producing its particular commodity.

The problem of variable production coefficients over the course of time can ordinarily be handled within the existing model by means of a periodic statistical revision (say every three years) of the empirical input-output table. Alternatively, if such periodic revisions are beyond either the financial or research capabilities of government statistical agencies and departments, planners might still be able to project future coefficients by using technological data from other slightly more developed countries or by merely extrapolating from past trends. Finally, the Ministry of National Planning should always keep a close watch on certain strategic industries like transportation and construction where any change in production techniques or
purchasing patterns can have significant repercussions throughout the entire economy.

Unlike the problem of variable technical and capital coefficients, the problem of multiprocess production possibilities within individual industries presents serious analytical and practical difficulties for the application of most simple input-output manipulations. Suppose, for example, that there exist two alternative methods of producing the same output, say cotton textiles. The textile industry would consequently have two different sets of technical coefficients. The problem then arises as to which process to use under various circumstances.

This is a problem not only of equating inputs with outputs to ensure the internal consistency of the development plan, but more importantly, a problem of choosing the optimal production process, that is, the one that can yield the same total output while using relatively less of whatever resources are available. In short the existence of alternative technologies and resource "constraints" requires use of more elaborate methods of linear programming and the formulation of a combined input-output linear model known in the literature as "Activity Analysis".
REFERENCES


